Instability and Transition of Fluid Flows

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Lecture No. # 16

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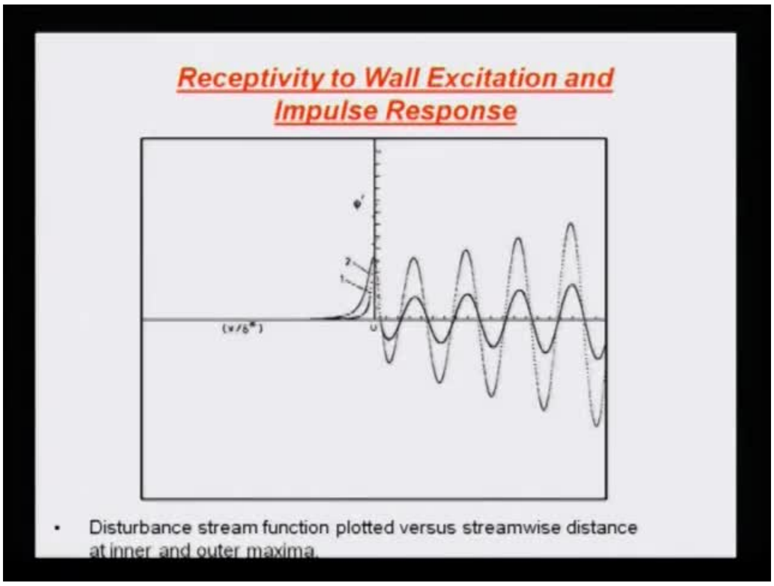

So, in the last class, we were continuing our discussion on receptivity of boundary layer to wall excitation, in the context we will look at the impulse response that is, exciter is place to the origin which is vibrating at a pre-defined frequency. As a consequence, you create a kind of a wave train which grows in space; so, this is the corresponding picture of your spatial instability problem. So, this is what we had doing, and we saw that the disturbances are stronger near the wall, and as you go away the decay; so, this is the one solid line is for the outer point, which we call as the outer maximum, and the dotted line, I represent the inner maximum of that boundary layer.

So, this is what we obtained, and we pointed out that, there is nothing new learnt, if you look at far away from the exciter, this is the kind of information that you have would have obtained by using spatial stability theory, only difference you can see is that, you did not have to make any assumption, whether its belongs to spatial stability or temporal instability problem.

All if do here is just simply provide a excitation at a fixed frequency, and see what happens correspondingly, because it has the asymptotic path, that is the waves, those are far away from the location the exciter, but you have a near field solution in the close vicinity of the exciter itself; this is the receptivity theory gives you stability theory has no clue, how to give it, and we notice that the solution does not appear discontinuously, it appears continuously, there is a bit of a upstream penetration, and then, it just latches on to the asymptotic part of the solution.

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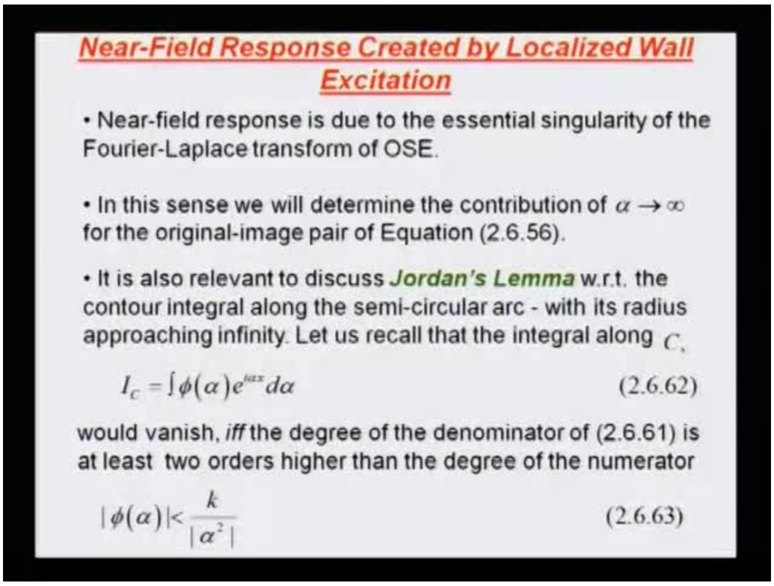

So, as we are interested, now to look at the special attribute of the receptivity theory in create explaining the near field response due to this localized wall excitation, then we did talk about the role of essential similarity, that is the point at alpha equal to infinity, and essentially, then what you are trying to do is find out what that phi of alpha is when alpha has going to infinity.

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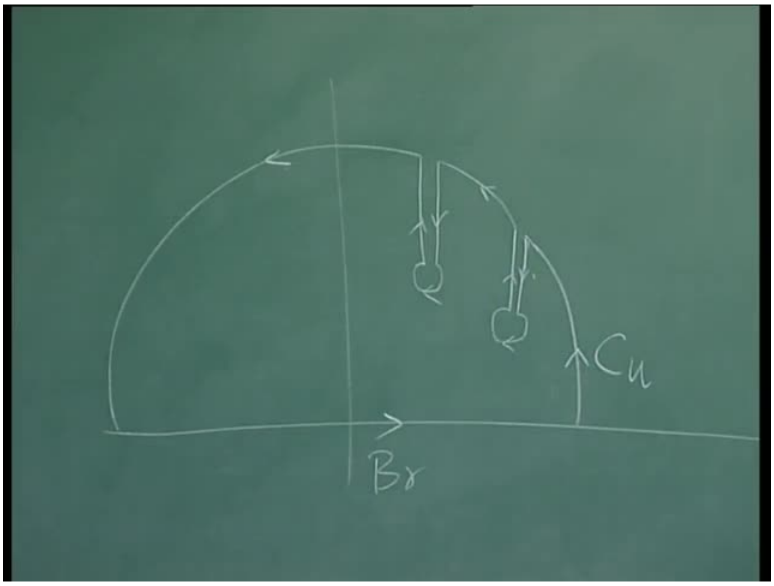

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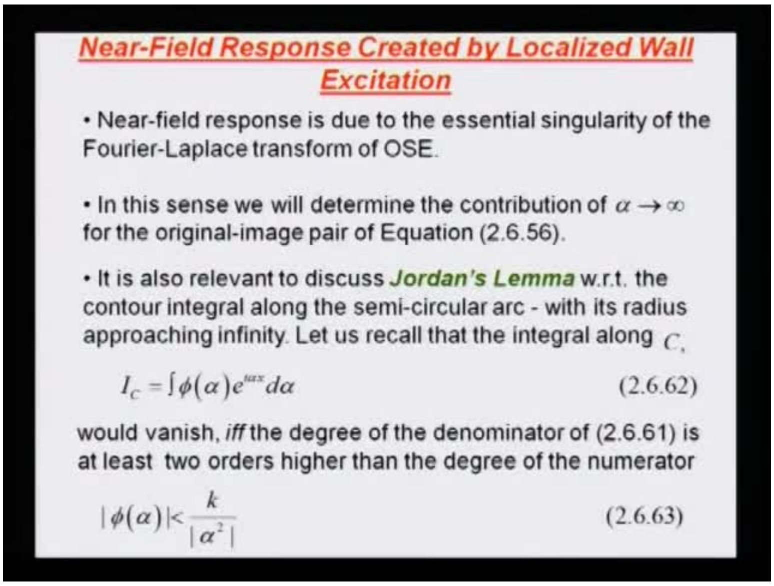

In the context, you do have to revisit Jordan's Lemma, which says that the contribution coming from the semi-circular arc with be vanishingly small, as the radius of the arc goes to infinity; so, this is the kind of contour that we talked about in the presence of one similarity, and let us say, put couple of all similarity, and we are talking about this kind of a thing. so, basically we have this cut, and we do perform the Bromwich contour integral supplemented by this contour in the upper half, which goes like this, so we have seen. what we get in terms of Cauchy's integral formula and theorem, and we are talking about this part, the contribution coming from the semi-circular part Jordan's Lemma claims, that if phi of alpha has this kind of a structure, where the denominators order is at least two degree higher than the numerator, then this will not contribute anything.

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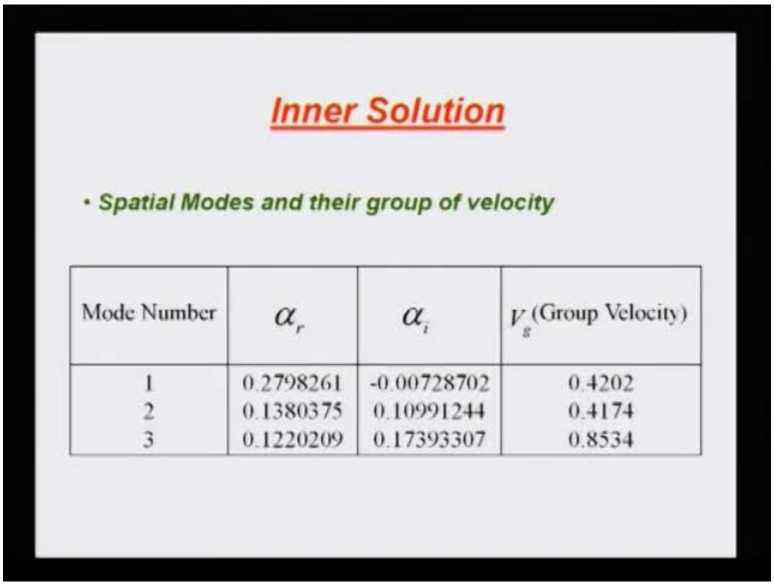

So, we decided to take a look at that aspect, and if phi of alpha indeed has this property or not, and that is why we began also we noted that, there is something more to this whole story, that when you look at the spatial modes, along with the group velocity for this particular case of Reynolds number of 1000, and this non-dimensional circular frequency of 0.1, you only get three modes, that is it, and that makes us a little worried, because how may, I going to explain a very arbitrary input disturbances, in terms of three modes allowed, it is impossible task. In that context also, this near field study becomes vital, because what is the absolute near field definition, it is the point itself, and that is where we are applying a delta function.

So, if I can somehow support a delta function, I know, what I can explain anything as a combination of delta functions, any arbitrary functions could be shown in terms of the distribution of deltas.

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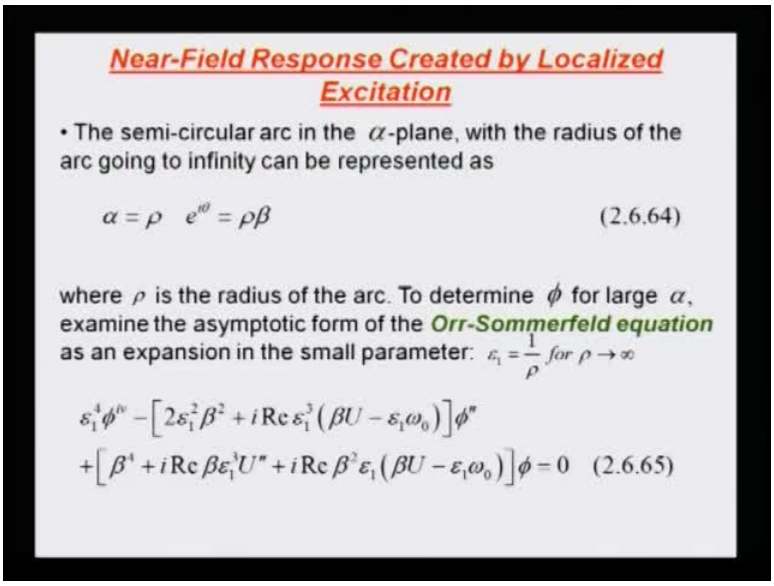

So, that is what we notice that for this specific case, we studied, we have one unstable mode, and two stable modes, all three modes propagate downstream, and let us, then, see what we get about the near field, and that was, what was the most interesting part that we started, and in studying the near field, we talked about couple of theorems (( )) theorems, and also the properties of Fourier Laplace transform, and its duality property, what we find that if we are interested in finding out the response field, near the exciter, then it is necessary, that I would look at the point alpha going to infinity, because that is the property we know Fourier Laplace transform.

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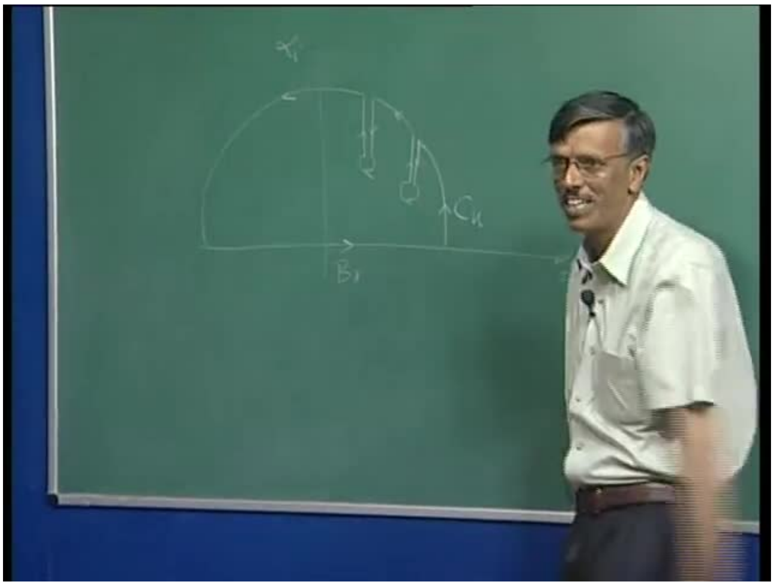

Anything that is localized in physical plane, in the transform plane, it is all pervading and vice versa, so if I am interested in finding out the solution, that the immediate neighborhood of the exciter, I would better be looking at the point at infinity, and the point at infinity is interesting, because if I plot it in the alpha r alpha i plane the point at infinity is basically the circle of the radius going to infinity, this is something we must appreciate.

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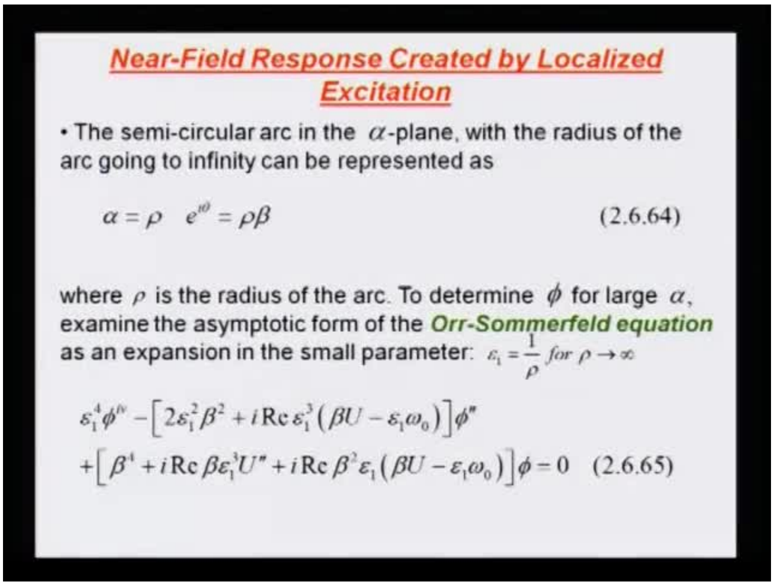

So, along any such location, I could represent the wave number in terms of the radius, and a phase path which I may call e to the power i theta or I can write out a shorter notation called beta; so, beta as beta real and beta imaginary…

Now, you have this expression for alpha in terms of rho and beta, we can look at specifically the solution of Orr-Somerfield equation, for the specific case when the radius of this arc goes to infinity, that automatically brings in a small parameter epsilon which is nothing but the reciprocal of the radius, and once I decide to do that, plug that expression alpha as equal to beta by epsilon 1, plug it in to the Orr-Somerfield equation, and I immediately discover the singular perturbation nature of this equation, because the highest derivative is multiplied by the smallest parameter, and this is essentially the issue of all singular perturbation problem, like the original problem propose by prenatal himself, then how did this problems circumvented, we stretched the layer by introducing a new length scale.

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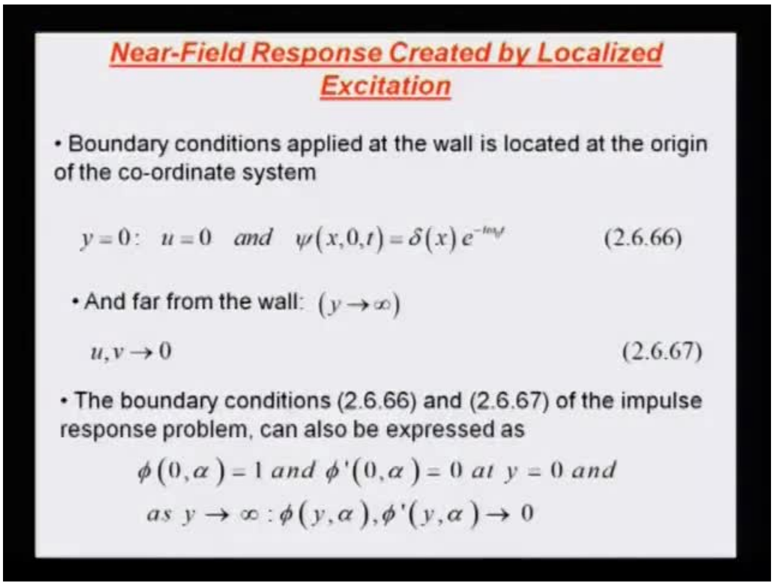

So, what we had, we discovered the boundary layer which was thin, now we are going to do the same thing, we are going to discover the boundary layer of the Orr-Somerfield equation, that is something rather interesting, we are going to solve that equation subject to this set of four boundary conditions required for forth order Orr-Somerfield equation two at the wall, and two at the far stream, and in the alpha plane, this four conditions become this, and positioning of the exciter causes this phi of alpha to be equal to 1, that is the property of a localized function, that it excites all wave numbers with equal intensity, and that is the essence of impulse response studies.

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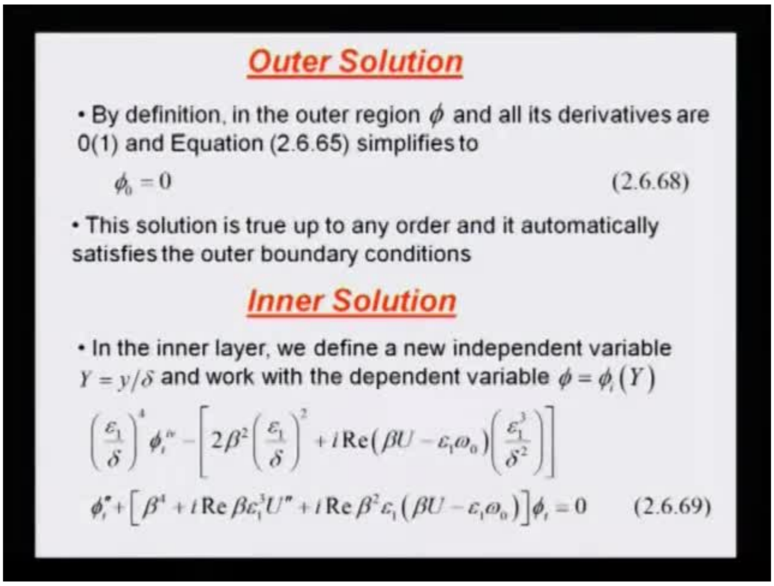

So, we are talking about discovering a boundary layer of Orr-Somerfield equation. So, if I am doing that, that I should have two layer structure, at least one should be the outer layer, another be the inner layer; in the outer path, by definition all the quantities of any order of five could be of same quantity of order one, and that brings out the solution to be equal to 0; so, this is quite a sort of revealing solution, that we do not have to do much in terms of fixing the outer solution itself, it is a trivial solution.

However, the inner solution is quite rich, and to explore what this inner solution is what you need to do, you need to again stretch it, and that stretching is via the scale delta. So, we call that as capital Y as a new independent variable, that is y over delta, and then, we discussed in the last class, that if I now introduce the primes with respect to this capital Y, then automatically this 1 over delta, and its various power shows up the fourth derivative will bring us 1 over delta to the power of 4 and that what is you are seeing.

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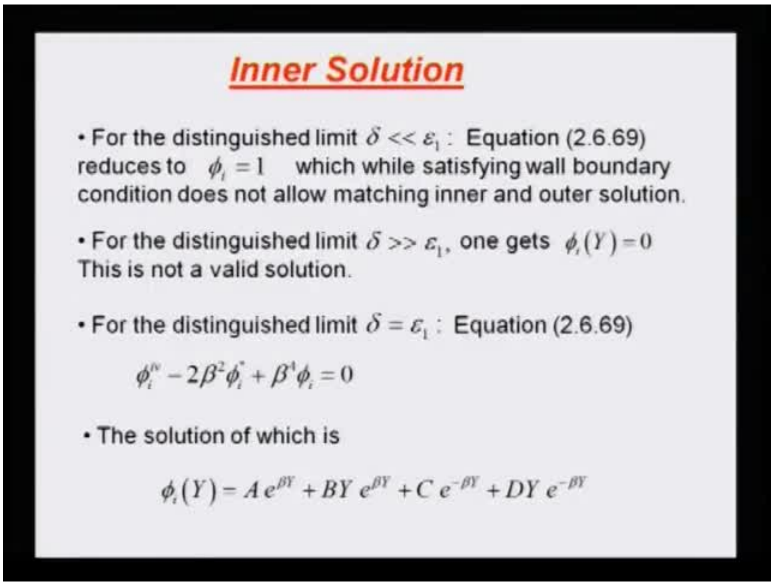

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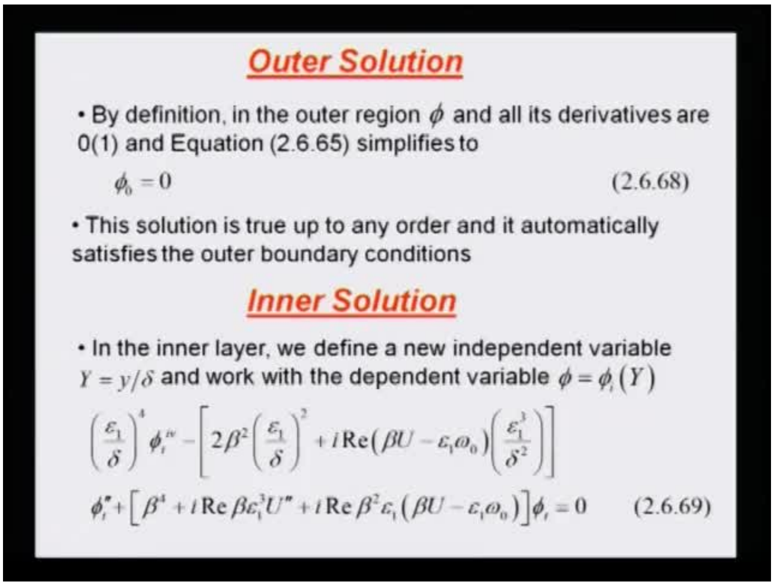

This is the story of similar perturbation theory, you will have do what is called as matching, matched asymptote, you have to do, you have to see in what way this epsilon 1 and delta appears, for example, we can think of a very general characterization, a general characterization would be something like this boundary layer thickness of this Orr-Somerfield equation, delta is very much smaller compare to epsilon 1, if I do that, then immediately I can see what happens, if we just look back in the previous slide, if delta is very small compare to epsilon, then what happens, this quantities I could multiply the whole thing by delta to the power 4.

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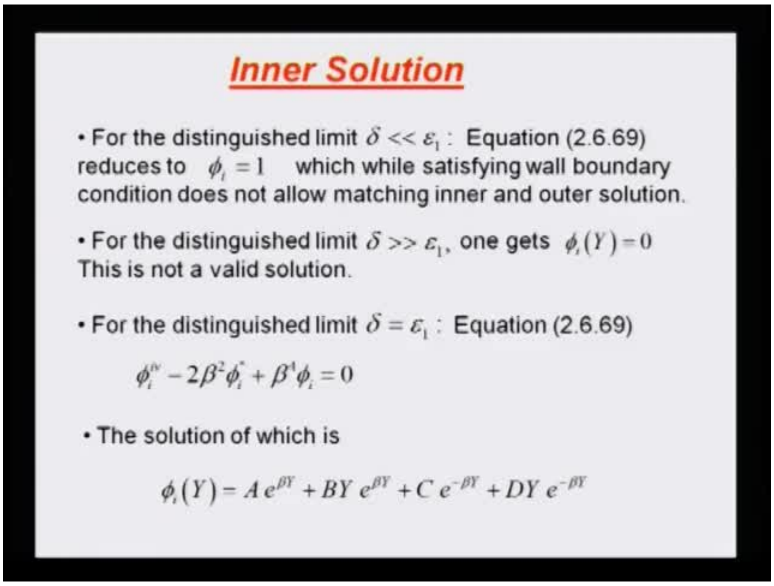

If I do that, then this will come like this epsilon 1 to the power 4, and here everything will be multiplied by some power of delta; since delta is sub dominant compare to epsilon 1,it is only this term, that will survive, if that term survives, because everything else goes to 0 5 4 equal to zero, then you can show it in terms of a polynomial, you will have a plus b y plus c y, and d y square, and so on, so forth, and then, we could use those four boundary conditions, and see what do we get, you will found out that the only possible solution is phi inner phi i should be equal to 1.

Of course, satisfies the wall boundary condition that is what we have done; however, this will give you a sort of a very funny situation, because your delta is going to be vanishingly small, so you have phi i equal to 1, and then rest of (( )), it is 0, so it does not really match smoothly, there is no smooth match between the inner and outer solution outer solution is supposed to be zero. So, we have one, and then, immediately zero, so that is continuity is not very mathematically rigorous, and correct we do not accept that as a solution.

Now, in other cases, for example, delta could be significantly larger compared to epsilon 1, and if I do that, and look at that equation the only solution that is feasible is this phi y is equal to 0, and this is a trivial solution your outer layered solution, outer solution is zero, you have nothing.

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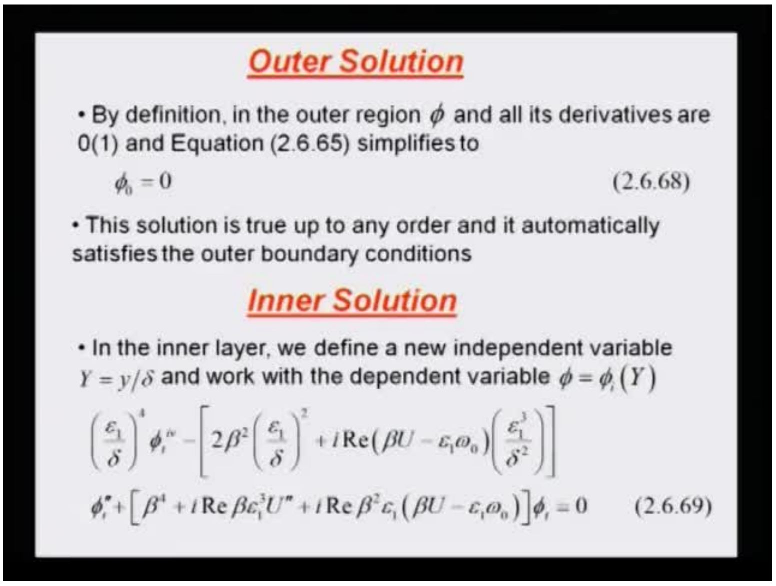

If you now again take a look at the governing equation, there are other possible pairings, for example, we could have epsilon 1 equal to delta, this is one order, one quantity this part is order one quantity, what about this part, this will go to zero, because this is I can write it as a epsilon 1 square by delta square that is order one times epsilon 1, so epsilon 1 going to zero, so that will not survive. So, from here, I will get 2 beta square phi i double prime, these all those terms will disappear, all though you will get beta to the power 4 times phi.

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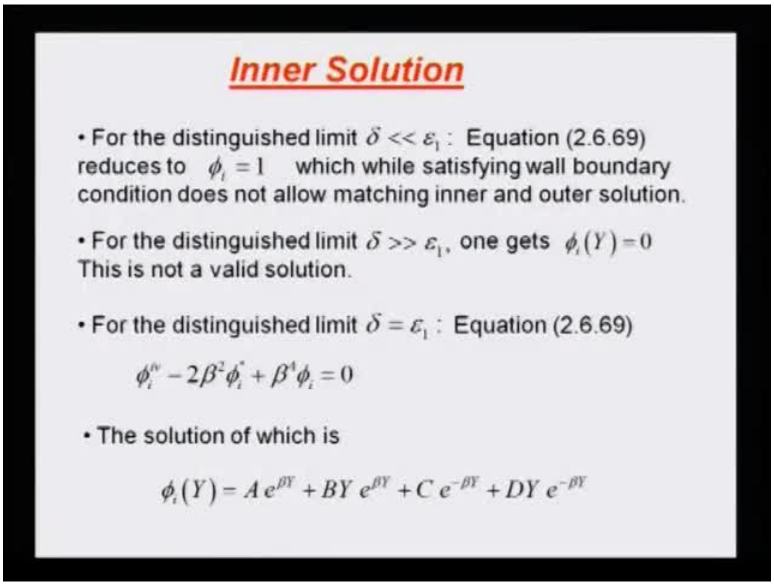

So, you get this term, you get this distinguished limit governing equation, when delta equal to epsilon, and now, you can see that, you have repeated roots plus beta and minus beta as a characteristic exponent, so one of the solution would be A e to the power beta y, the second one would be B Y into e to the power beta y, then the third solution is C e to the power minus beta y, and this now you have this four constants capital A, B, C, D, and you can fix those by satisfying the boundary conditions, and we find that depending on beta real, whether it is positive or negative, to get these two solutions which satisfy those four boundary conditions.

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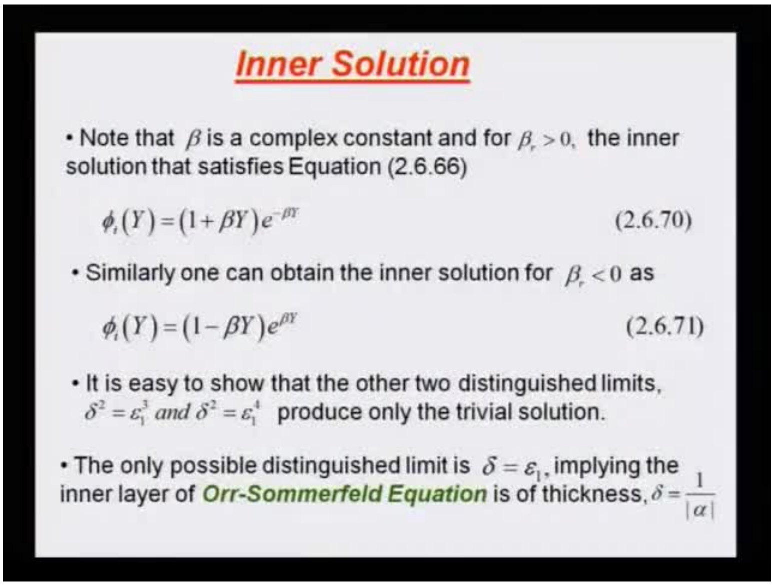

So, if you are on the beta r positive beta r positive, means, what you will be in this path because alpha i is rho times beta rho is real beta real means, real positive is this side, beta real negative is in this side; so, on this side, you have this kind of a solution, and on the left half plane we have this kind of a solution.

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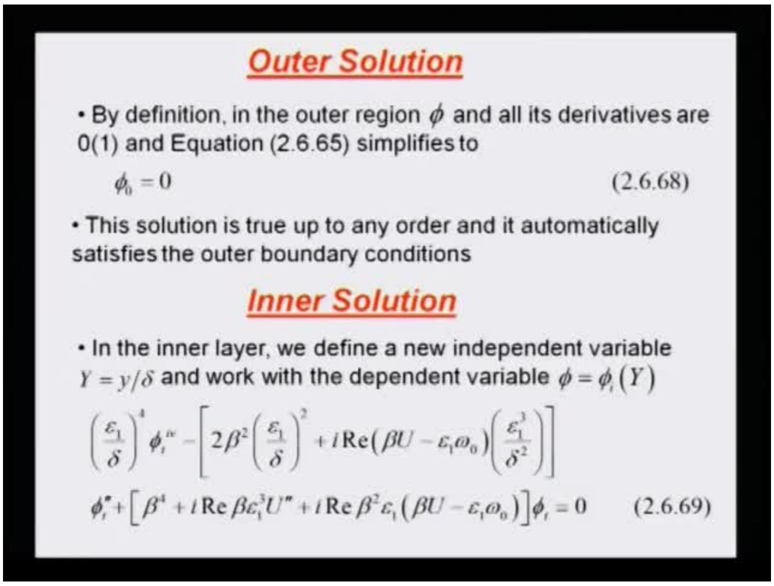

Now, in doing singular perturbation theory exhaust all possible combinations, that you can think of, see for example, I can note some kind of a distinguish limit appearing here; this is something like your epsilon 1 q by delta square, we can investigate, that I will not do it, I will leave it for you to explore, and see that they do not produce nontrivial solution.

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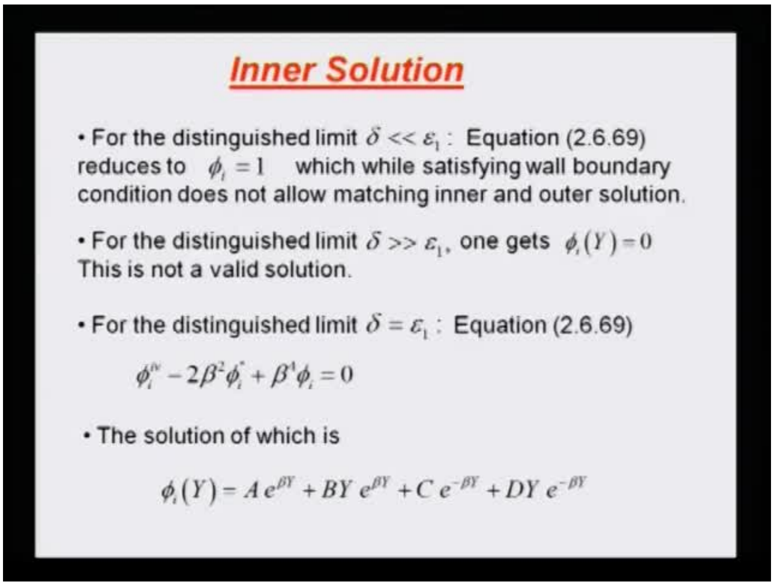

So, the only distinguish limit that we can talk of is essentially delta equal to epsilon 1, and the solutions are as given here, and I that is what second last point, we just now talk about that we can have this following two distinguish limits, where delta square equal to epsilon 1 cube or delta square equal to epsilon 1 to the power 4, and for you to show that, these two distinguish limit leads to trivial solution. Now, what are we found out, that is a very interesting thing, we have found out Orr-Somerfield equation as a boundary layer, which thickness is given by delta, and delta is equal to epsilon 1, and epsilon 1 itself goes to infinity.

Now, this is all very nice about talking about mathematics physically, what happens if I try to excite a flow, can I excite any wave number think of it physically; physically can we excite all wave numbers, what is the wave number related to can you relate it somewhat with the energy content. You see what happen, I have a uniform flow, I am creating an excitation, what is a consequence I am creating a stress field, that strains the fluid on the stream rate is proportional to the curvature of the stream lines, and if I create very large wave number, I am creating waves which actually distort very rapidly in small region, and alpha going to infinity, this strain rates are going to go to infinite value.

So, mathematically we can talk about alpha going to infinity, but in reality, we will have a finite limit on the maximum value of alpha; so, that would also give you some kind of a finite thickness of the boundary layer of the Orr-Somerfield equation.

So, it is not like epsilon 1 going to zero, and delta also going to zero; so, again we have some kind of a discontinue solution, here of this two branches, and at the end of the boundary layer, it is go to zero, that is your alter solution is right.

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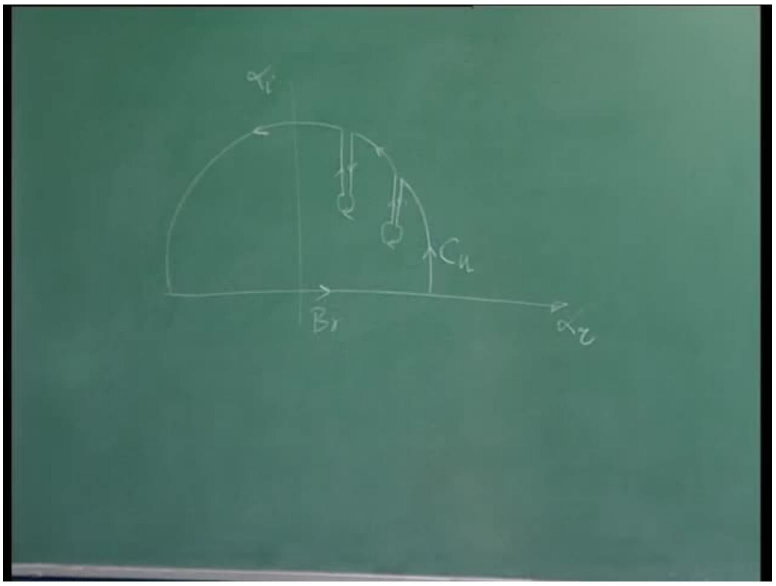

So, aesthetically this is true those of you may have had some exposure to turbulent flows, you may have heard of people talking on (( )) length scale, there also the people tell you the same thing, that you cannot sort of excite any arbitrary length scale, because eventually the strained rate will match to the desperation, we will be created via exuding energy as the heat.

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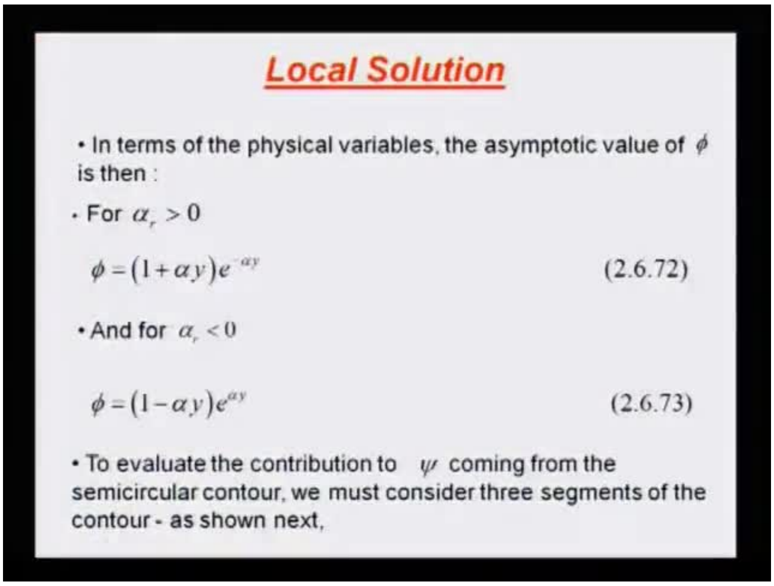

So, that fixes the length scale alpha max; so, here also we are noticing the same thing it is not tubular flow, it is a laminar flow, that we are talking about finite energy disturbance, so we cannot have alpha infinity, we have finite quantity and the solution is what we are talking about.

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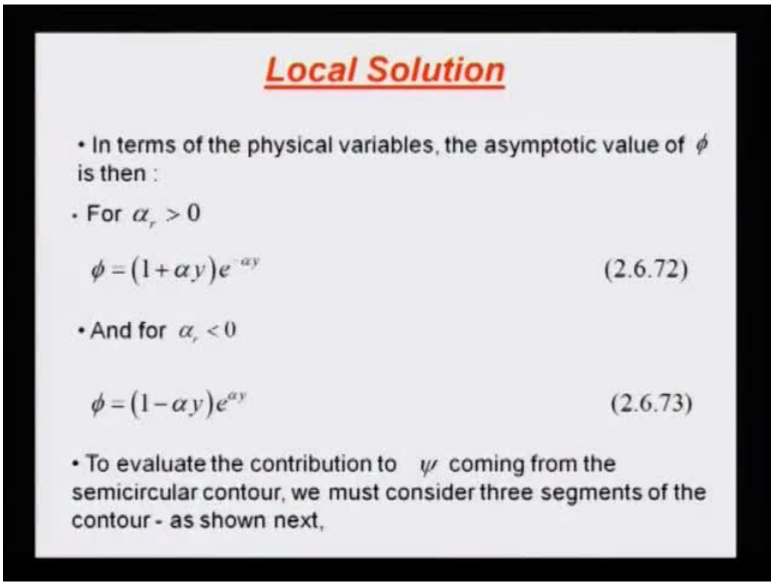

Now, if we talked about in terms of rho and beta, so if I now switch over back to alpha, this is what those two solutions are this 72 is the solution here, 73 is the solution there. So, you have now the expression of phi along this semi-circular arc, life cannot get better than this. In associate, what we said, we wanted to investigate if Jordan's Lemma is correct, if it is not, what is the contribution? You can very clearly see Jordan's Lemmais not necessarily correct, because you can write it in a form of numerator by denominator, but it is not directly evident that Jordan's Lemma would be valid, in fact, what we can do, we could evaluate the contribution coming from the semi-circular arc.

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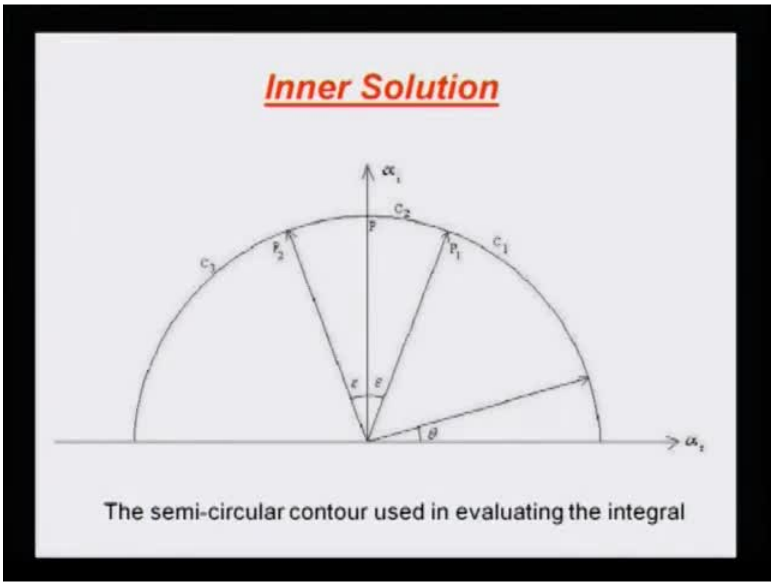

So, you have a task at hand, because your semi-circular arc is continues, but the function is not on this part, we have 1 plus alpha y e to the power minus alpha y, on this side 1 minus alpha y into e to the power plus alpha y ;so there is a kind of a discontinuity at this imaginary axis path, so to avoid that kind of a problem what we do is fragment the semi-circular arc into three paths; the first path is going from real axis to up to a point here, this point P1 and P2 are defined by this small angle epsilon.

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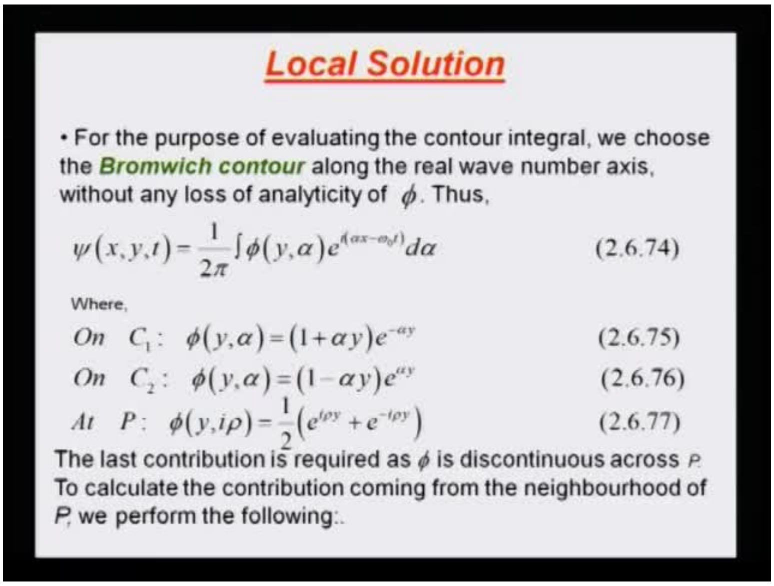

So, in the limit what we will do, we make this epsilon go to zero; so, this P1 and P2 will approach P. so, what we will do is we basically would find out the contribution coming from this part, coming from this part and coming from middle part.

In the middle part, your solution is discontinues that you know how to handle it, you take the average of left and right hand limit, that we are quite aware of what to do. So, basically done we are performing this integral, and on C 1 the right segment, we have phi y of alpha given like this, 1 plus alpha y e to the power minus alpha y; on the left part left half plane, we will write 1 minus alpha y e to the power plus alpha y, and at the point P, we will have to take a limit average of left hand, right hand limit, that we can very clearly see that alpha at the point P is becoming purely imaginary, and that is your i rho.

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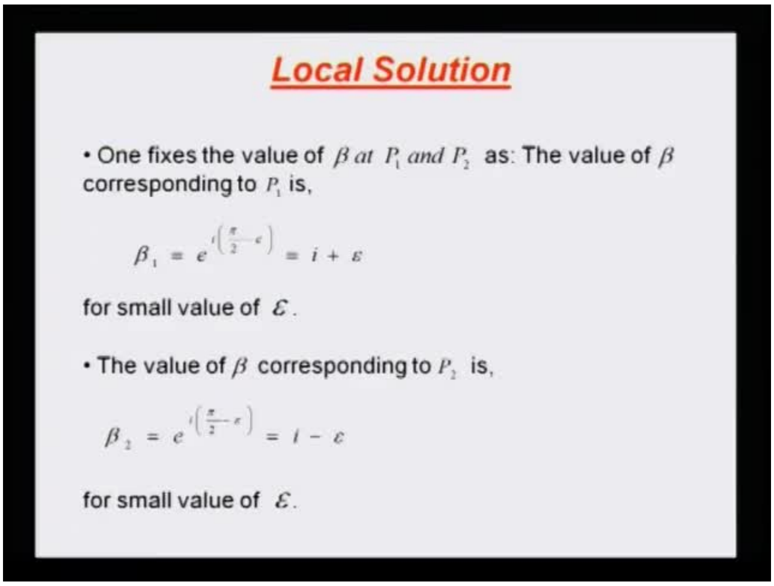

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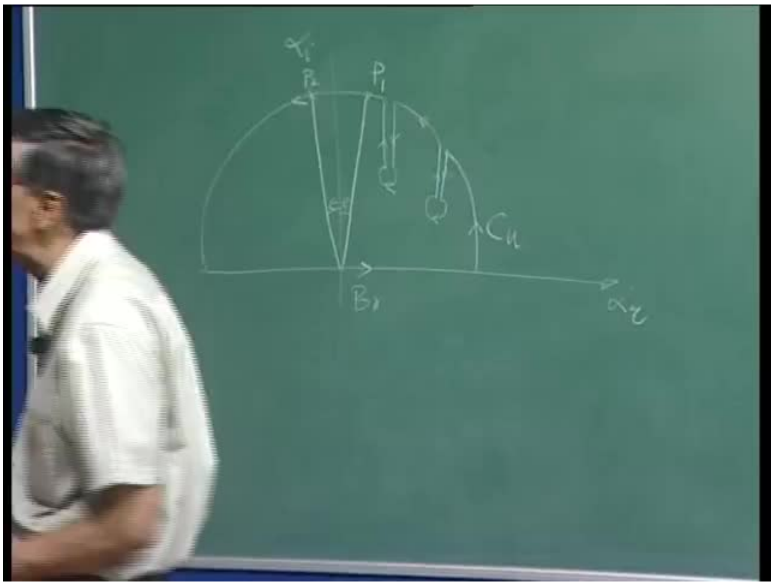

So, you get this part this is cos of rho y, and what kind of a function, we talking about so disturbance which goes zigzag's all the way to infinity, it is artifact of the particular part, and this is anti-stokes line, we will not talk about anti stokes line, but let us keep it in mind, that we can perform this integral, all though the function may behave a little un physically, but its contribution can be worked out, and those two points P 1 and P 2, that we are talking about. Here, by positioning this is your P 1, and this angle is epsilon, and similarly, I have another point here P 2, and that angle is also epsilon, so we can find out the corresponding value of beta on this side, you have theta is pi by 2 minus epsilon.

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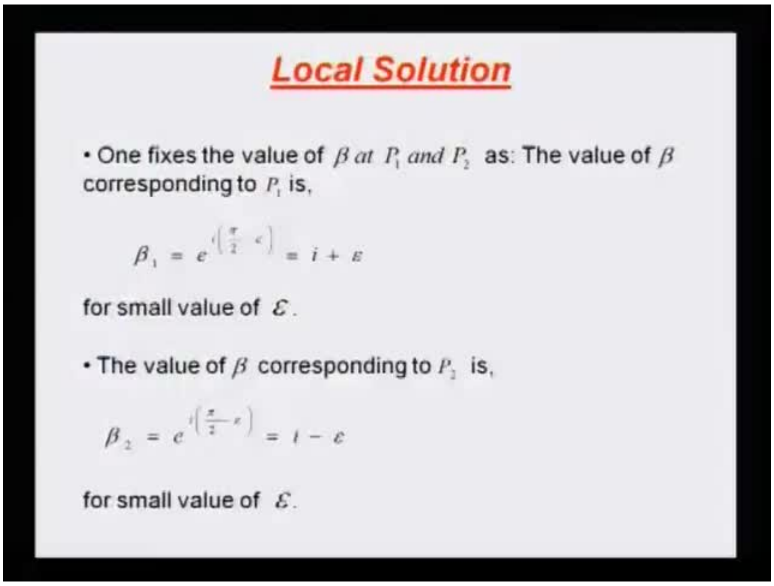

So, if I do that, beta 1 can be written as i plus epsilon, and similarly, for this point P 2, I get beta 2 should be equal to e to the power i pi by 2, there should be a plus sign here. It should be plus that will work out to i minus epsilon, so for small value of epsilon, we can fix those phase beta 1 and beta 2, and then, we can go ahead and calculate various components the first component, I, one would come from this point to P 1.

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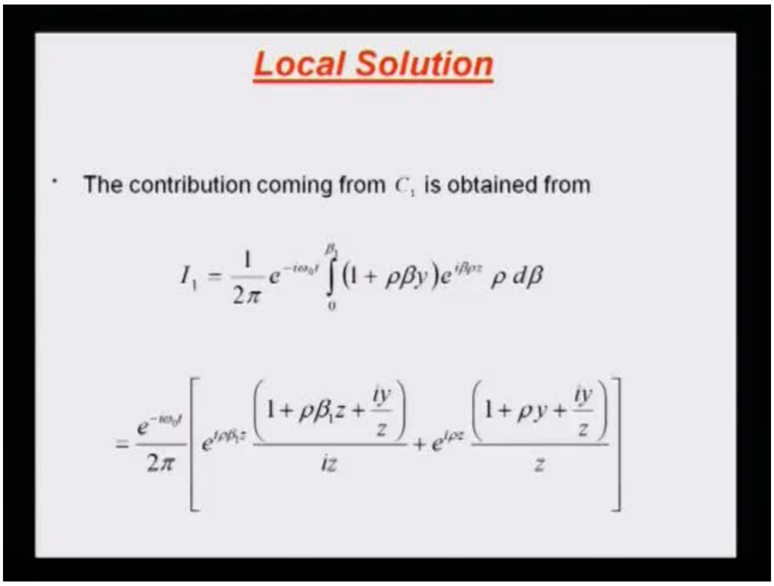

So, 1 over 2 pi e to the power minus i omega naught t performing the integral from 0 to beta 1, and this is your value of alpha 1 plus rho beta y e to the power i beta rho z, and d alpha becomes this d alpha is nothing but rho times d beta, and then we perform it it works like this, its works out like these two expressions with this complex variable z that is equal x plus i y, that is your i 1, and you can calculate the contribution coming from P 1, P 2, this path again work it out it was cos rho y, and this is what we are going to get.

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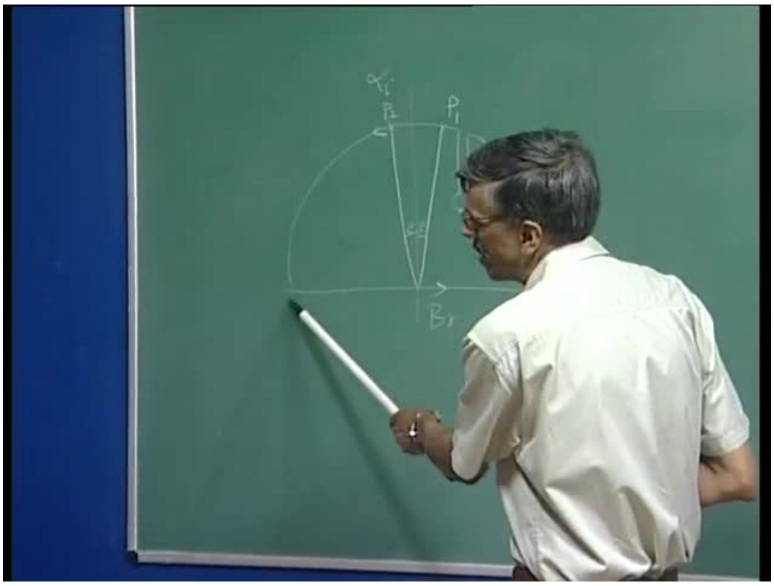

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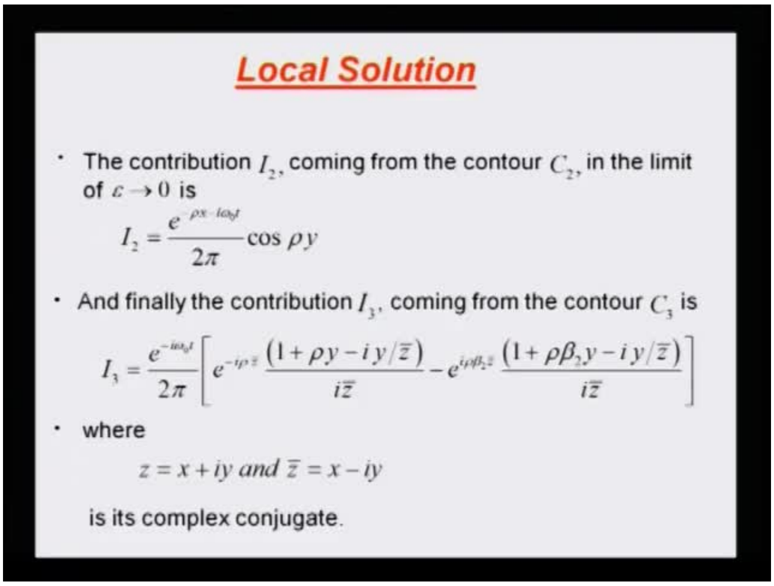

Finally, the contribution coming from P 2 to this path, that we are calling as i 3, and that would work out in this particular fashion. So, you can work it out, and see there will be in terms of the complex conjugate of z, that is x minus i y.

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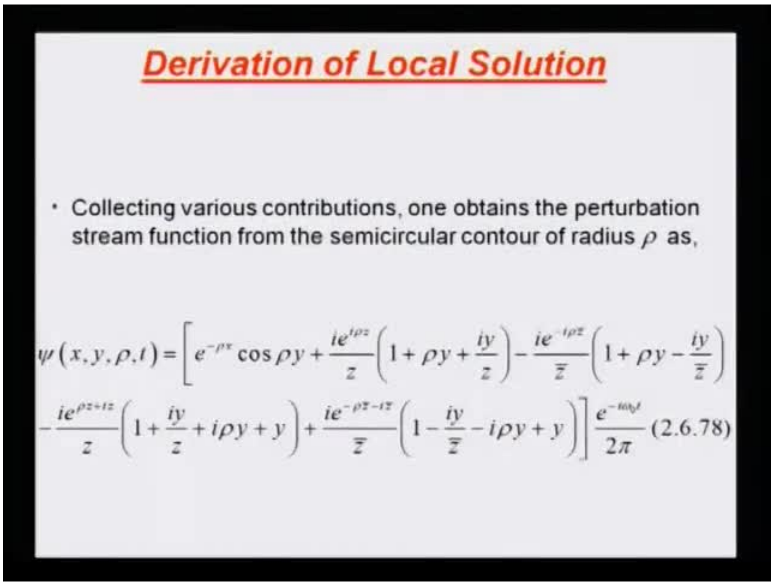

So, it is easy you have the expressions for the contributions coming from three different parts of the semi-circular arc collate it, and this is what you are going to get. We got to remember that we are looking at the specific case, when rho goes to infinity, and this function, of course, if we try to got it, and try to plot it, this will look hoariest, because (( )) fantastically fast varying phases.

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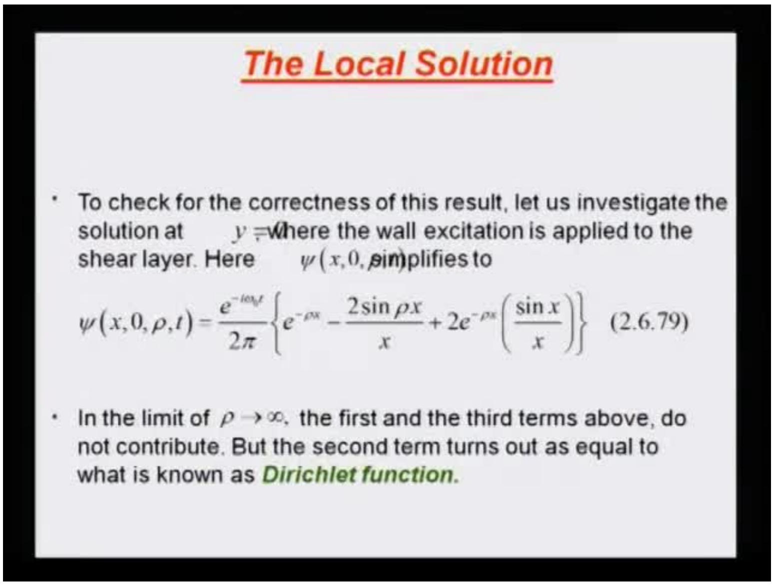

However, what happen to the solution, when you go to the point, where we are exiting that is at x equal to 0 y equal to 0. So, if we do that what we get? We get an expression like this psi of x at y equal to 0, it is given like this, this simplifies to this. Now, what you notice that very interestingly enough that the first, that the third term will have associated e to the power minus rho x.

Now, if rho goes to infinity, this goes to zero this goes to zero, so sin x by x is the sin function varies from 1 to 0, and so and so forth, so this is bounded, what about this, this is a very interesting function, this is a function that is called a Dirichlet function. You know, there are many approximate way of expressing delta function Dirichlet function is one such representation, and in the limit rho going to infinity, this path gives you delta x.

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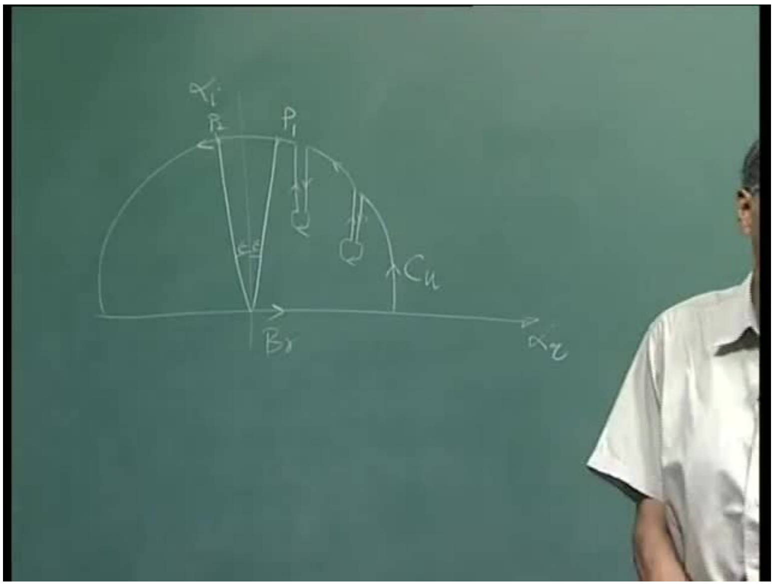

So, this is satisfying what does it tell us as a solution, we are not going to talk much about, whether they are the right solution or not, but what we could do is, we could construct a solution analytically now, and that solution shows that you can recover that delta function excitation, so what is interesting is that, if I take a boundary layer, and if I excite it at one point, the delta function is supported by the point at infinity.

This is very satisfying, this is the property of Fourier Laplace transform, if I have looking at, what it is going to be, and that is precisely what we got, and if you now recall the figure that we drew at the beginning of this class. We saw the solution at inner and outer maximum, and that the more, we come closer, let us go back, and then, you will see what we are talking about.

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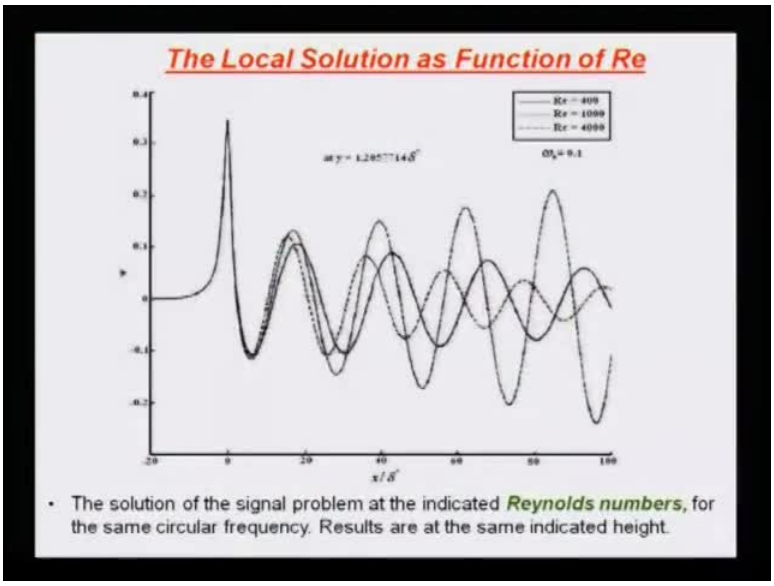

I think, they are is this solution, let us look at this is a similar solution, a solution which is plotted at one point two times delta star, that is pretty much on the outer edges of the boundary layer, but if you keep looking at the lower point, then you would find that this curve will come here, and this peak will go higher, that is what you have seen, that as y comes close to the wall your response field peaks up becomes narrower, that is we are talking about representing the solution in terms of Dirichlet function rho going to infinity.

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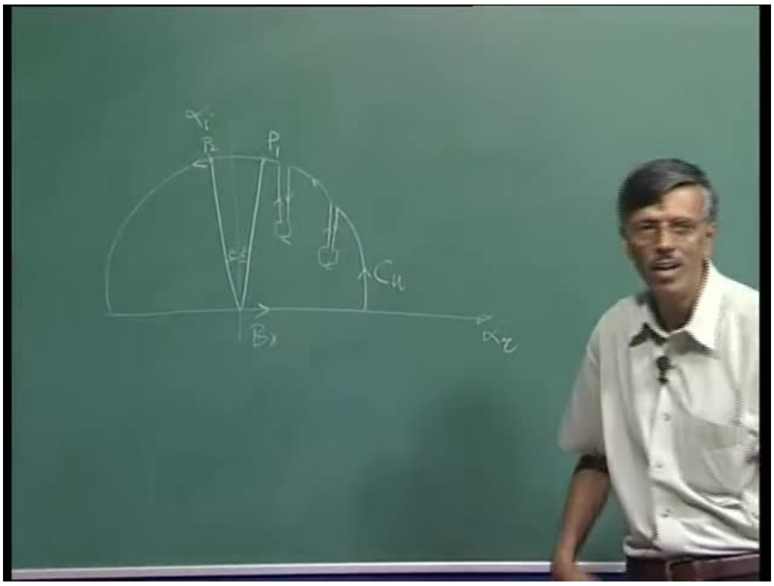

So, you see the beauty of it, what you saw analytically is also, what you recover computationally; this is all from the solution of the Orr-Somerfield equation.

How did you get this solution by just simply performing a contribution from this point to that point, I mention this point, that once we do this Bromwich contour integral, information about everything is embedded there, all those effects of those similarities are there, the effect of this semi-circular arc, they are all betting in that solution, because phi y of alpha basically gets that.

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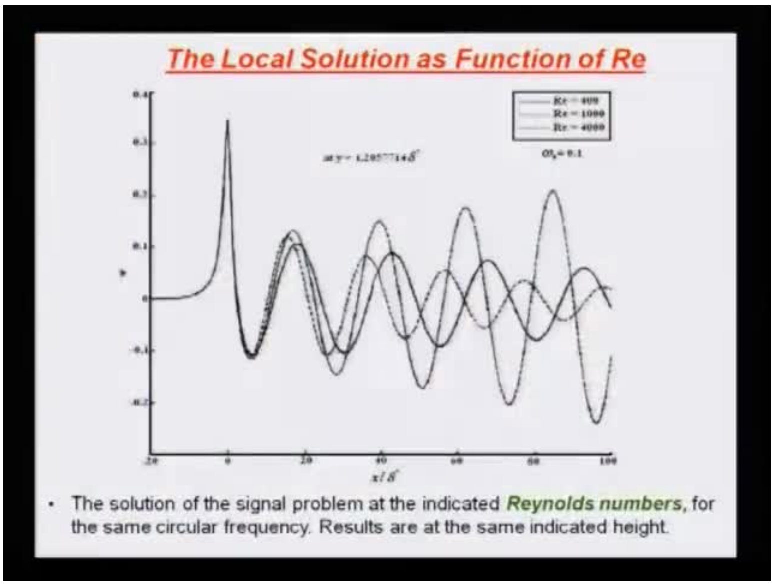

Now, we have crossed a major hurdled, now let us talk about little bit about the property of this near field, and here what you are seeing, solution plotted, response field plotted for the psi as a function of x over delta star for three Reynolds number 400, 1000 and 4000, why did we choose these three numbers for 400, what do we expect, it is below the R e critical.

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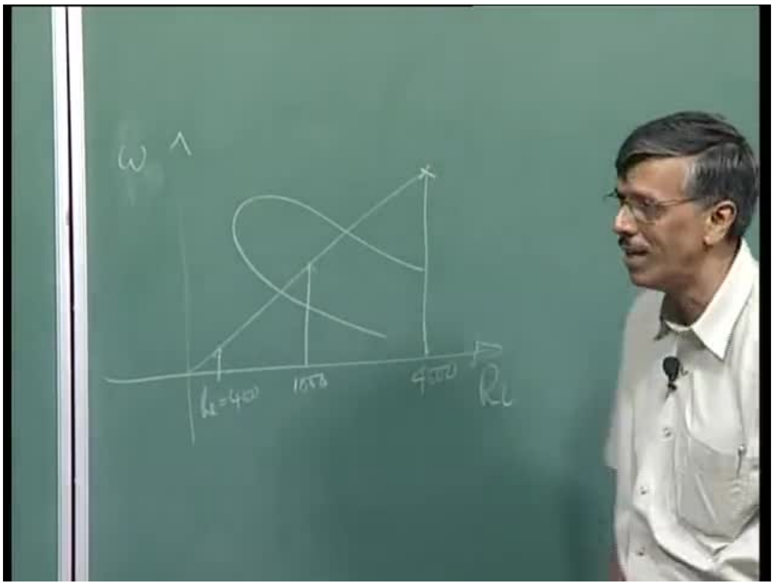

So, the disturbance should be damped, and this is that 400 solution, it is decaying that is that path. What about 1000, 1000 is inside that neutral loop, and you have unstable solution that is this solution is its growing, and when you go to 4000, if you recollect, got your, let us put it as omega, omega, and this is your neutral curve, and we are talking about omega equal to 0.1 solution. So, it should be something like this, so, 400 is somewhere on this side, so that is your 400R e equal to 400, and this is 520, so this is there, 1000 is here, so this is your R e equal to 400, and this is 1000, and 4000 is somewhere here, that is of again going to be stable, and in fact, that is so far away from the neutral curve that decay rate will be stronger, then what you have for 400, and that is what you are seeing that 4000 solution is here.

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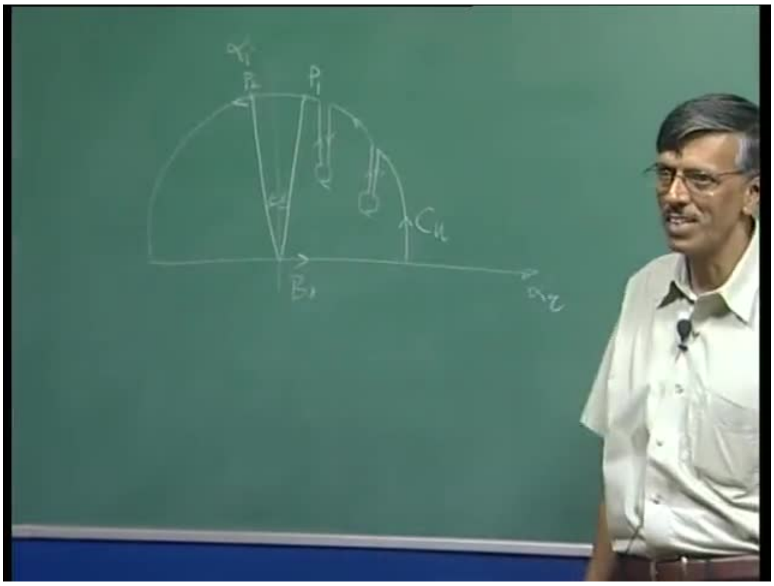

So, what this stability theory tells us, we also verify we see though the near field part of the solution, they all collapses with each other, it will not depend on Reynolds number, if you recall it comes from this contribution, and what is a governing equation, if you look at the governing equation, that we recovered there is given by equation 80, do you see a reynolds number appearing in that equation.

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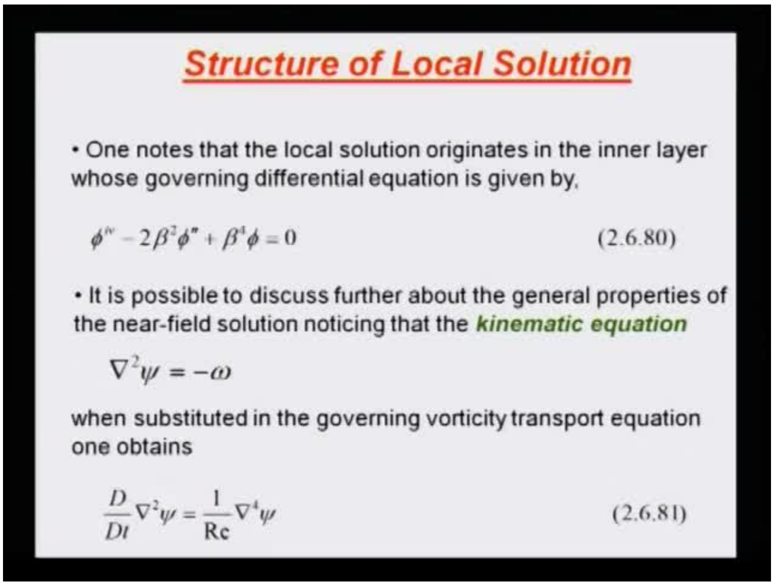

So, no wonder, so this is a indication of wanted to do what you wanted to see, and what you see this is a self-consistent result by itself, that is why, when you perform the

Bromwich counter integral numerically, you get the solution, and reason that your local solution does not depend on Reynolds number, we will talk about it further, because where is this term coming from, to understand where this term is coming from, let us look at their Navier stokes equation, the only grid of fluid mechanics, everything should be included in Navier stokes equation.

So, we wanted to find out where from this term comes, so let us look at the Navier stokes equation, I have purposely written in terms of stream function and vorticity. So, this is the kinematic definition of vorticity del square psi equal to minus omega you plug it into a vorticity transport equation; so, vorticity transport equation is d omega d t is the substantial derivative should be equal to 1 over R e into del square omega, so if I substitute omega equal to minus del square psi, I get this.

So, here we have a Laplacian on the left hand side, and here we have a biharmonic, now if I multiplied by R e, then I get this, then psi biharmonic equation, if I write it in terms of Fourier Laplace transform, what would it be this, this will be this. So, what you are seeing is very revealing that the local solution, for which the governing equation is this comes from the right hand side equal to zero, and that means, locally the solution behaves like the flow as R e going to zero that is it.

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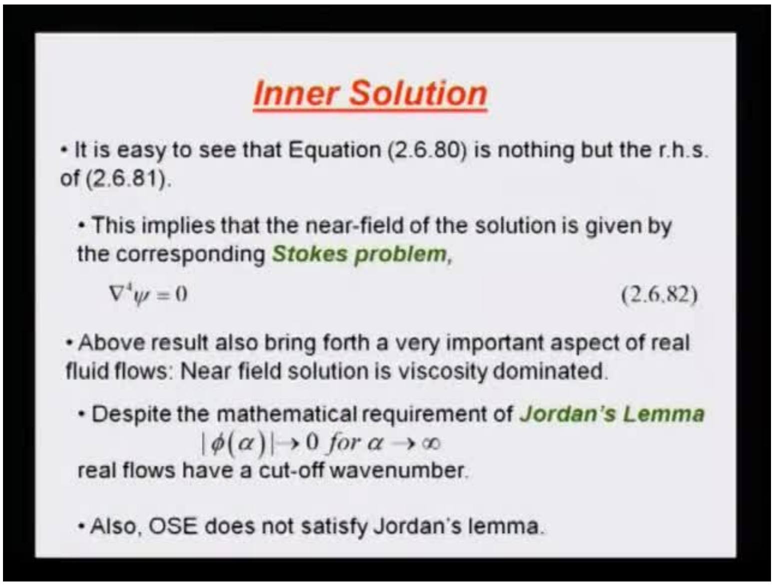

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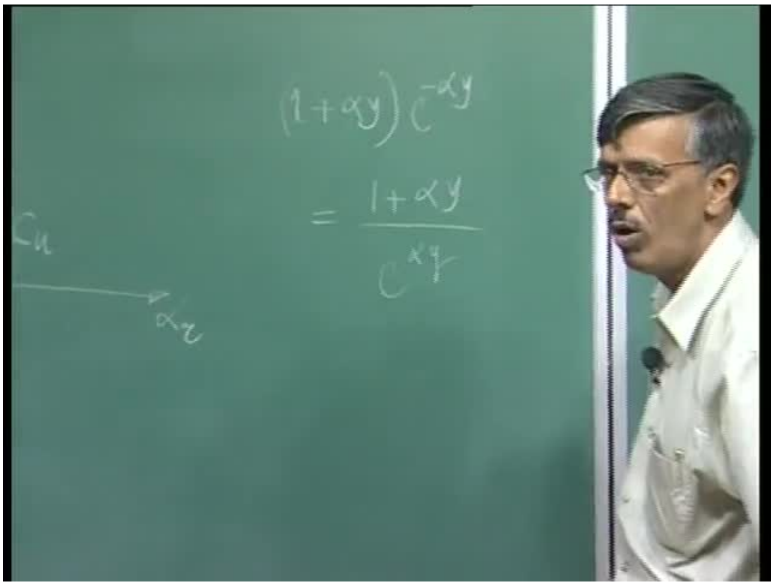

So, that is the nature of your local solution, local solution is given by stokes solution, that is a stokes solution R e going to 0. So, we are seeing this thing very elegantly, it is not due to some assumption, this is all coming from that singular perturbation analysis, the inner solution reduces to the stokes problem that is delta 4 psi equal to 0.

So, Near field solution is viscosity dominated, because R e goes to zero, this is something that we would like to see, and we have already commented on this aspect, that requirement of Jordan's Lemma, that phi of alpha should go to 0 for alpha going to infinity real flows have a finite cut-off wave number, and this more or less is self (( )) explanatory tells you that Orr-Somerfield equation solutions, may not necessarily directly give you that exact formulation, that we are talking about, because we have the solution like this(( )), let us say for alpha real positive this is like this, so this I can write it as 1 plus alpha y e to the power of alpha y.

So, can we say that denominators order is more than two orders more than the numerator what we are saying that it does not satisfy the condition of Jordan's Lemma, and this was not proven before this was what we have done it, we preserve this, and today, you are seeing it being presented in front of you that Orr-Somerfield equation admits an analytic solution, in the element alpha going to infinity, and once you obtain that you establish that your phi of alpha does not satisfy Jordan's Lemma.

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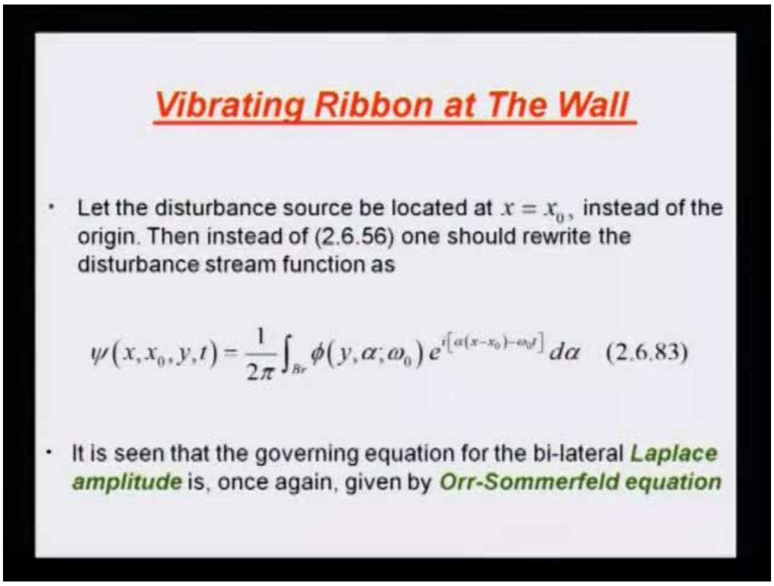

That is calling it as Jordan's dilemma, because when we did not know, it was a problematic thing, whether to accept it or prove it, and we try to prove it, we ended up disproving it, so that is the dilemma. Now, let me talk about some side issues, that they are not trivial, they are not unimportant, we have obtained the basic unit process of the flow excitation, that is by using a delta function imagine, what was the experiment done by schuhbauer and skramstad they vibrated a ribbon.

What is the width of the ribbon, it is not a delta function, it is a finite width that is why we want to talk about instead of having a delta function, if I have a finite width. Now, we realize this we commented also, if I have the solution for delta function, I can take it convolution of that solution, and I can represent any arbitrary function, so we can do the same thing here.

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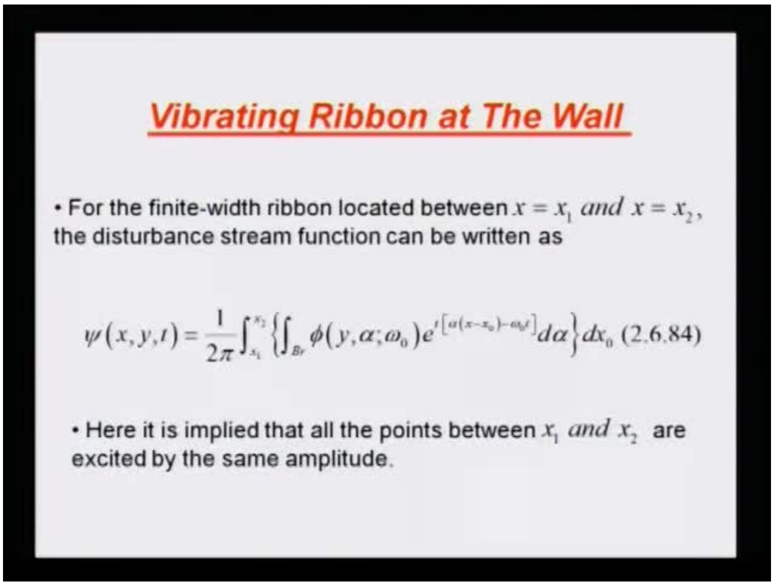

Suppose disturbance source is located at x equal to x naught, then this will be the solution, and again I can substitute it in linearized Navier stokes equation or apply a parallel flow approximation, so and, behold I get back the Orr-Somerfield equation for this phi, so for any point this is the governing equation remains the same, what I need to do is, that take that unit solution, where the exciter is at x naught, and then integrate over if range of x naught going from x 1 to x 2.

Let us say, I have got this impulse response that is what we shown you, so far the solutions, then I integrate it over this x naught from x 1 to x 2. So, what are we assuming here that each and every point is excited with the same amplitude, so this like the whole thing going up and down, so that is one way of doing it.

Analytically it looks neat, but if I tell you to go ahead, and try to do it for the solution of Navier stokes equation, you will (( )), why, because of this discontinuities at the end of the strip at the end of the strip, you will have zero, and then a finite value, and that will excite all possible alphas, and numerically that is a disaster story. So, people use different kinds of boundary conditions excitation for the solution of Navier stokes equation, which will show that, as far as analytical solution or numerical solutions, where Orr-Somerfield equation is concerned because we should be able to do it.

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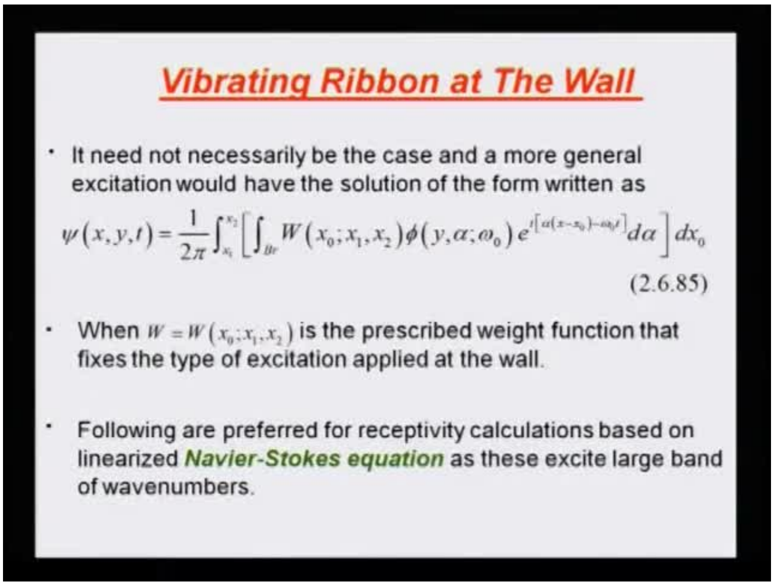

You are going to get solutions for each excited location, I will get some impulse response, and the total solution would be a convolution of all this, and we have already familiarized our self with group velocity, how did we get the groups of waves, when two neighboring signals very close to each other interacted constructively, and destructively at different locations, wherever it did I constructive the thing, we got the wave packets. So, here also same thing will happen, however what we could also do is the problem, that I was talking about solving Navier stokes equation, you may not like to give equal weightage. So, you could perhaps multiply some kind of a weight function gives something, which may smoothly take it to zero at the end of the strip and board.

So, that is what one can do, one can multiply by W, the weight function which will be function of permanently x naught, but it is define in the limit x 1, x 2, so we could define some function here and can work on it.

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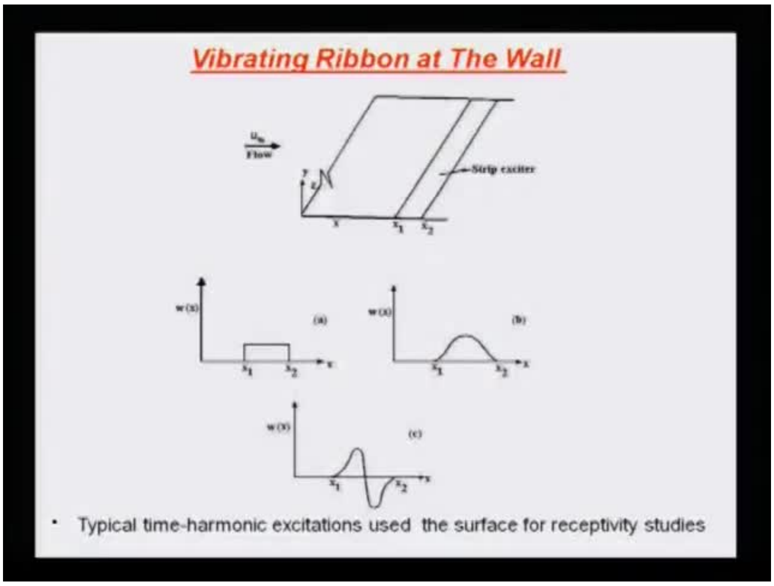

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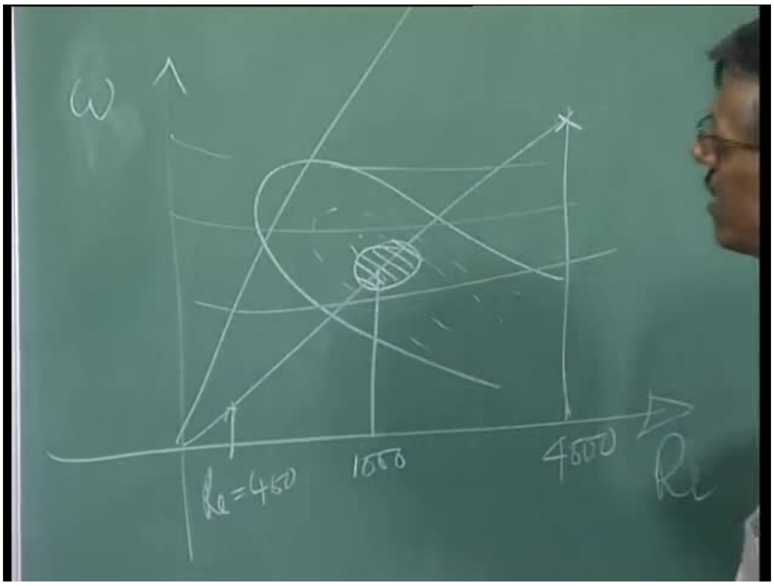

Now, I told you that from Navier stokes equation, we do not want this heaving motion of the strip, instead we do not want to do this in a Navier stokes solution, we want to do something different, where at the end, we should have amplitude equal to zero, this is one we way of doing say Gaussian, and we call while talking about Fourier Laplace transform, we mentioned a property of Gaussian. So, it is a Hermitian function, there it just a self-reciprocity, if I do it in the x space, like this in alpha space also, it will be like this, and we have commented upon against such activity, because this signal that you are creating is a band limited signal. So, if I have doing this kind of a calculation, and this relates to a range of alpha, if you remember what we talked about earlier on, that alpha i equal to draw like this, and what about the alpha r contours they go like this.

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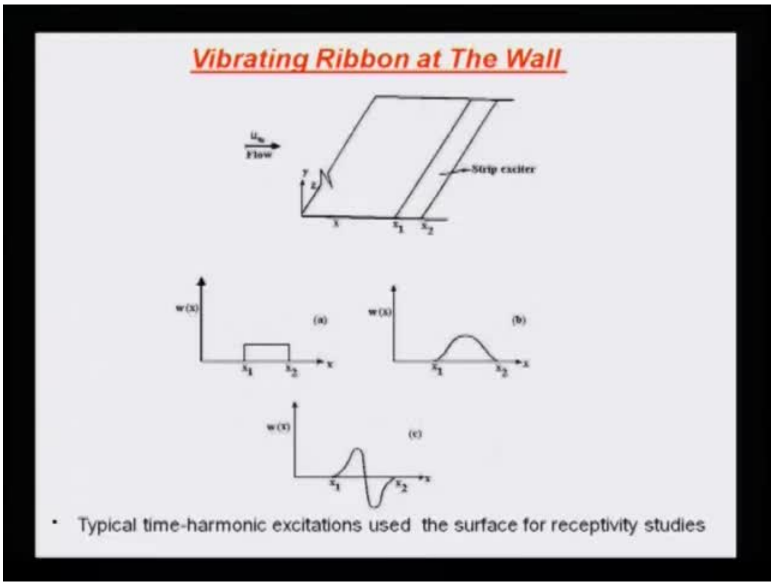

Now, if I create a Gaussian function over as narrow range, that could only excite a narrow band of alpha r and alpha i, so but this one, if I take some, suppose this is the range, that we are trying to excite, but that would also require to get the unstable solution, I should be doing it along this frequency; suppose, I do a frequency like this does not do anything, so there is always a worry a problem, that if I take this, I may be doing something which is not very revealing.

So, this is not what, one wants to do it, this is what one can do with the help of Orr-Somerfield equation, this is what one can do, and one does. What is this, this is called a simultaneous blowing section strip. What is so good about it, if I do it like this, half the part is positive half the part is negative the sum is zero, and if I am creating a kind of a ribbon with this simultaneous blowing section strip property, then at each and every time half of the ribbon will act like a source the other half will act like a sink.

So, you do not have net added mass, there is a something, also you must ensure that when you are computing, you should not violate any of the conservation principles, and this particular excitation satisfies the conservation of mass at each time, because this is the scenario at any time, this whole thing is harmonically fluctuating at a frequency omega naught, so half the cycle this will remain positive, other half it will be negative, and this also will corresponding reflect at each and every instant, you are going to ensure there is no net mass added, in fact, you would see that in the literature, there is lots and lots of article people talk about zero net mass jets.

You know this is one way of people trying to control the flow, and you want to control a flow like this, you do not want to create a spurious mass generation or create a spurious sink, instead you would like to have this, and what is this in the limit, suppose x 1 and x 2 approach is 0, it is a double delta function as is very specific name, this is called the doublet.

So, doublet is nothing but the first derivative of delta function, and that is why delta function doublet behaves like a vector, it has a direction going from source to sink. So, these are the kind of things, that is one can talk about, when it comes to Navier stokes solution, you will use this, this all is little risky, either you have a very wide ribbon, but as I told you that stability analysis is nice, in the context of normal mode analysis, means, you study one alpha at a time, but in the receptivity context, we are performing an integral over all alpha, and it this phases are so nearby to each other, they can create a group.

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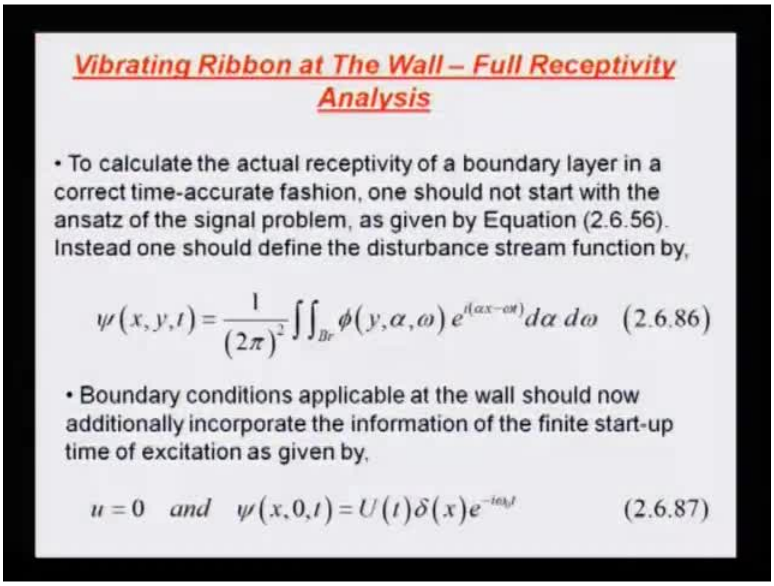

So, if I am trying to create a monochromatic wave, this is not a good way of doing it, whereas this one is, because you can really make a localized one, and you will be creating a just a very narrow width function, it will not interact with each other. I will give you some references of work, that we have done in the past, where we have actually studied this problem. Now, so far what have we done is, we have talked about so-called signal problem, we excited the fluid dynamical system at a frequency omega naught, and we also got the response has omega naught, that was our assumption.

Suppose, we do not make that assumption, so instead of doing a Bromwich contour integral in alpha plane alone, suppose we go, and explode the full receptivity analysis, we do not make any assumption of signal problem, that I will be performing integral also on omega.

So, I will have two Bromwich contour, one in the alpha plane, one in the omega plane, and then I will write down let say a stream function given in terms of this. Now, again we can go back, and do all kinds of analysis, but first of all, we will have to specify what the kind of excitation is we have given, and we will start our next discussion tomorrow's discussion starting from this; so, we will be looking the full receptivity problem.