Instability and Transition of Fluid Flows

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Lecture No. # 15

We will begin again after this break, so this week, so let us see what we have done so far; we decided that instead of looking at Eigen value problem, we would better of looking at the receptivity problem, the distinction being that in a receptivity problem, you prescribe some kind of a definitive boundary conditions. So, for the flow positive plot plate, what we decided to do is still look at the linearized Navier Stokes equation, because we are looking at small disturbances imposed, and if you look at that linearized Navier Stokes equation in the alpha omega plain, that is the spectral plane, then we end up getting Orr-Somerfield equation, that is what we have written here.

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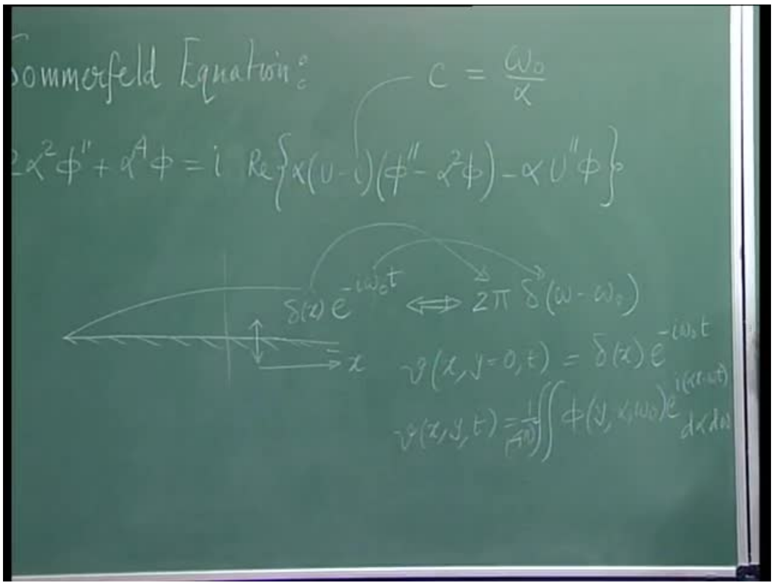

So, Orr-Somerfield is basically the unit around which all this discussions that follows, we will revolve around. So, what we have done in Eigen value analysis, we given all kinds of homogeneous boundary conditions.

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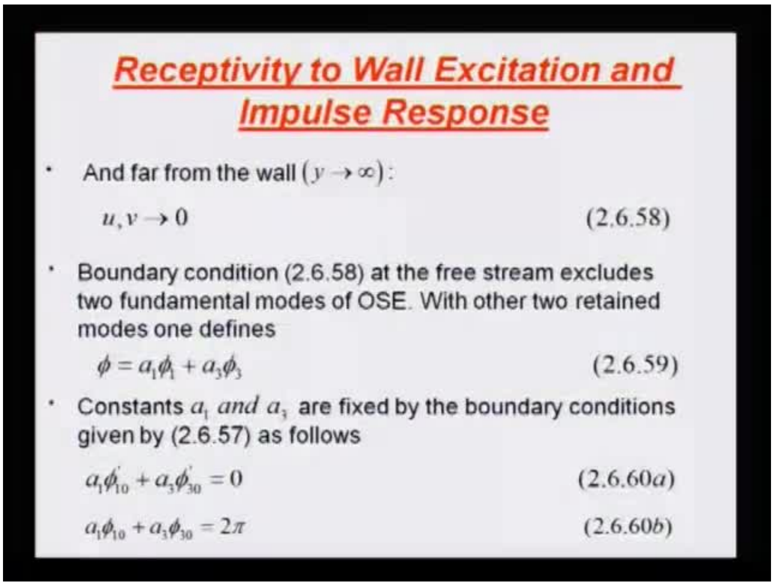

For example, this is quite a favorite condition to invoke that, if we are taking about disturbances, which remain embedded inside the shear layer, then if we go outside the shear layer those disturbances are not there, and that is the statement of equation 58 which says that, if you go far away from the wall, you have velocity components have to be 0.

Now, what we are talking about in this particular lecture and a couple of lectures to follow is we are going to study what is called as receptivity to wall excitation. So, we want to find out how your shear layer is receptive to disturbances, and what kind of disturbances this are imposed wall excitation kind of excitation, that we want to discuss. We talked it about it that is going to be the nothing but your those impulse response that we talk about; impulse response means we are going to excite the boundary layer at a fixed location.

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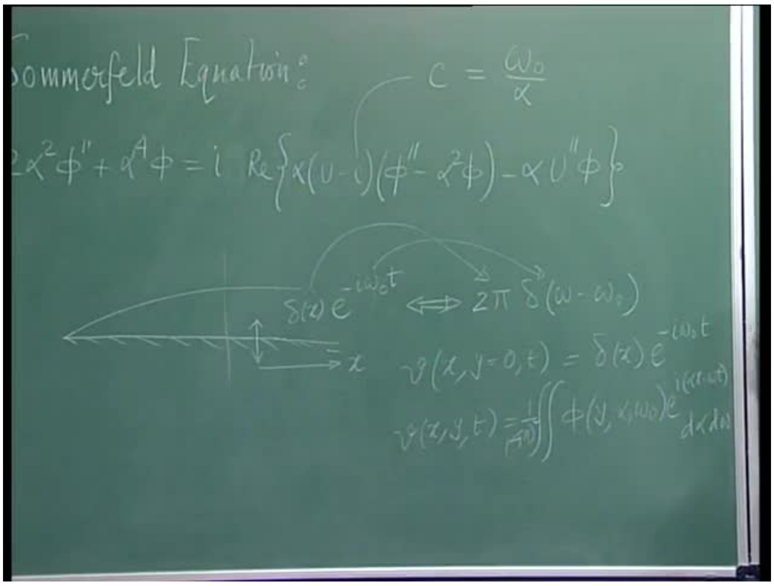

So, if we look at the schematic a bit, this is how it should look like that, we have a flat plate here, and over which a boundary layer develops, since the flat plate has a very sharp leading edge; so, we will have a boundary layer originating from there, and now what you want to do is we want to make use of this Orr-Somerfield equation, and I would try to tell you how we got this, we got this, because we made some vital assumptions, what was the prime assumption that the flow is parallel, that means, we are going to talk about that part of the flow, where the boundary layer does not grow any more.

So, maybe you know, I mean, we would exclude this part of the flow, where you can see a significant growth of the boundary line that part is not considered, but in this part, this works. So, if I want to study the property of the shear layer to wall excitation under the Orr-Somerfield a molar, then I would provide a localized disturbance, like the warm that we noted was performed by Schuhbauer and Skramstad.

Now, this is a delta function, so what I could do is I could actually fix a coordinate system like this, my x will start from here and y will go from here. See as far as parallel flow is concerned, there is no origin that supposed to be from minus infinity going to plus infinity, but it is a location of the exciter that imposes a coordinate system; so, we decide to have a origin there.

So, basically, then what we are going to do is create a kind of a mass source, a mass source which is given by a delta function, and then, this mass source is going to eject mass and suck mass half the cycle each.

So, we are basically going to give some kind of a time variation like this, and where is this omega naught here, this is here, see here is omega naught by alpha, then what happens, delta functions Fourier transform Fourier Laplace is 1, so that is what happens that, this has a Fourier Laplace transform which will be given by some 2 pi due to this part, the 2 pi is the normalization coefficient; so, it is, it is the relationship between direct and inverse transform.

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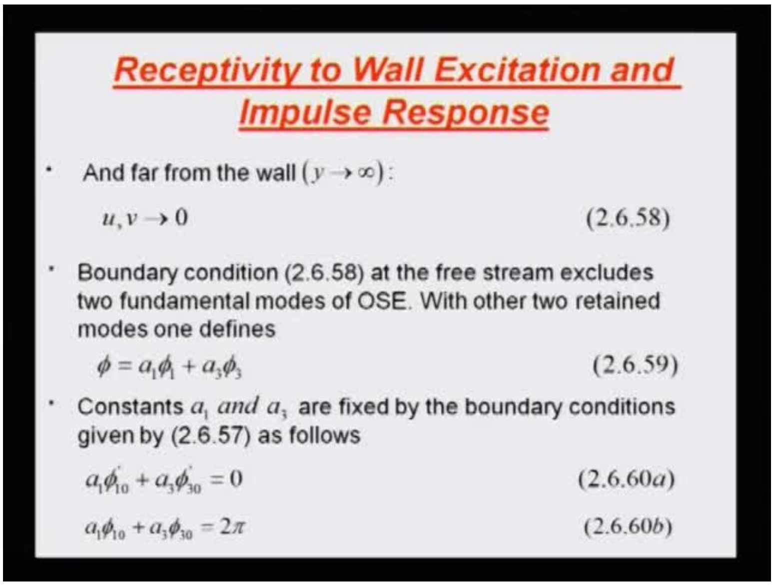

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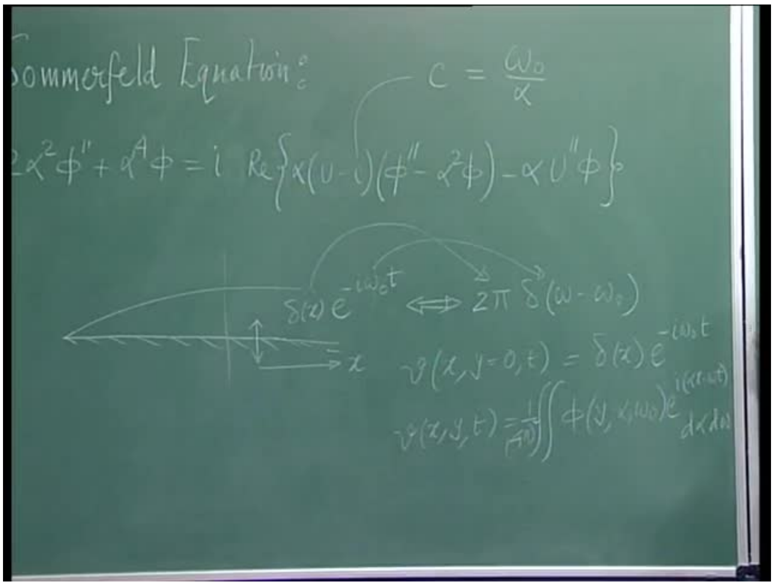

What about this one, this one we are assuming that is signal problem, that excitation is at omega naught only; so that is what it is, then what happens is talk about phi, what was phi, phi was the amplitude of the v velocity; so, this kind of a source here is equivalent to creating a wall normal velocity half the cycle, it is plus half the cycle its minus, and then, if I have v of x y equal to 0, and t is given by delta x e to the power minus i omega naught t.

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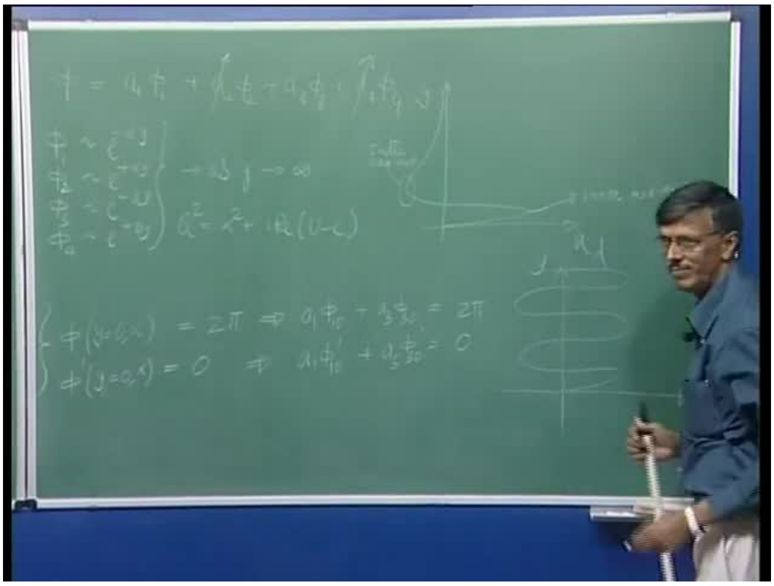

And what was this define as, if you recall your definition (( )) was like this; this was our definition, and we had a kind of 1 over 4 pi square, and this was the expression that we had. Now, if I club this two together what will it give, it will give me phi at y equal to 0, because we are applying it at the wall, and it will be also a function of alpha, and because of the signal assumption, so this will be nothing but simply equal to 2 pi.

It will be just simply equal to 2 pi, now what are we had so far, we had phi given as a 1 phi 1 plus a 2 phi 2 plus a 3 phi 3 plus a 4 phi 4, and what we found that phi 1 goes as e to the power minus alpha y, phi 2 goes as e to the power plus alpha y, phi 3 goes as e to the power minus q y, and phi 4 goes us e to the power plus q y, this is what we get as we go out of the shear, and you recall that q square was alpha square plus i Re U minus C.

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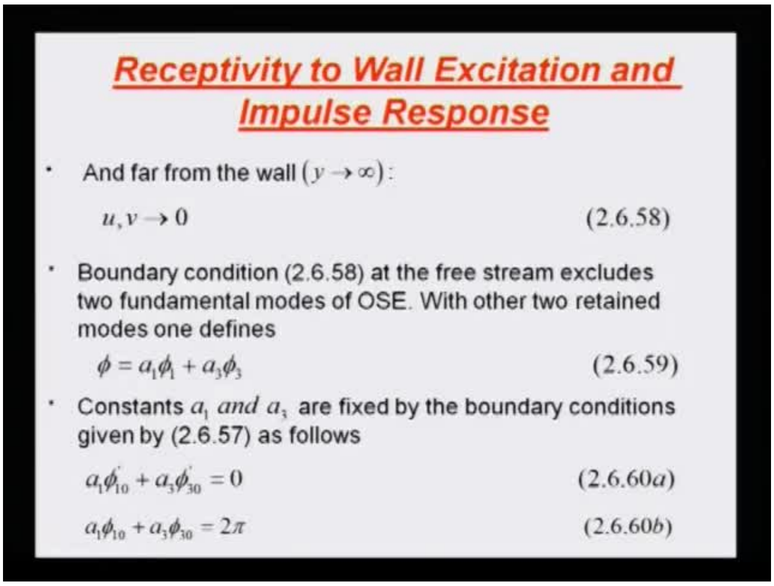

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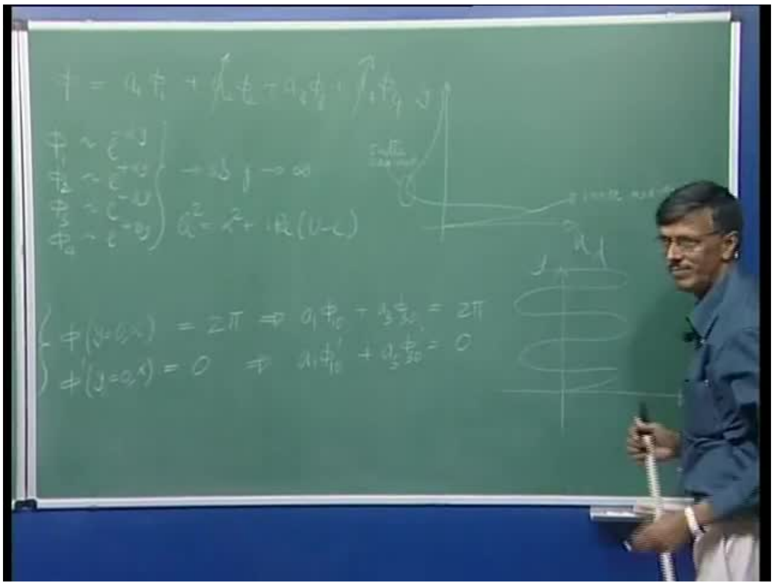

Then, those two conditions given by phi 8, equation phi 8 here would imply that, if alpha the real part of alpha, and real part of q are positive, then this must be equals to 0; please do understand, we are not saying phi 2 or phi 4 equal to 0, we are saying that called constant has to be equals to be 0, to satisfy those two vanishing conditions.

So, this, then would give you what this would give you a 1 phi 1 0 plus a 3 phi 3 0 should be equal to 2 pi, that is what we have written here, this equation the last equation is that. And what about the other condition, other condition is u equal to 0, what is u if you recall the mass conservation equation gives us what, mass conservation equation we did get that i alpha f is equal to phi prime, so if f is equal to 0, then phi prime has to be equal to 0 that you have done. So, basically we need to also have phi prime at y equal to 0 for all alpha must be identically equal to 0, and this gives you this.

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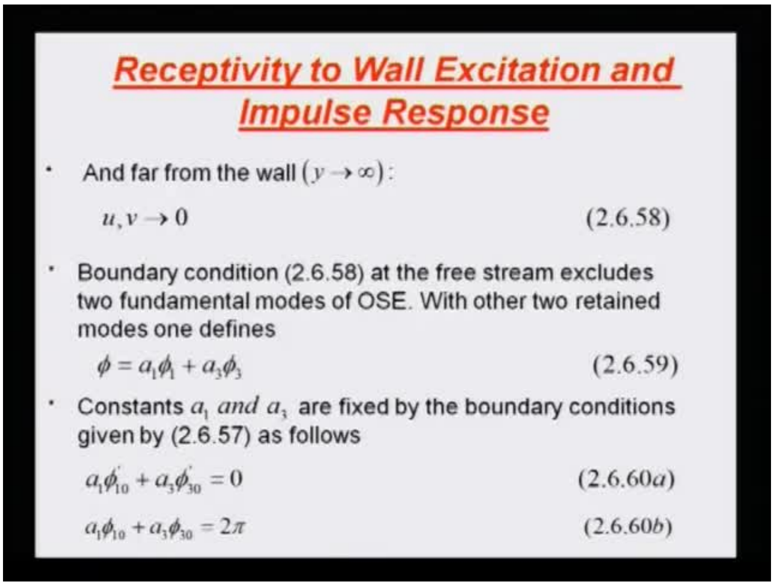

So, that completes your description of receptivity, so this is your definitive excitation provided at the wall, which is of this kind that translates to this two boundary condition. Now, it is more definitive in the sense, we can now fix the value of a 1 and a 3, by solving this equation, this was not there in the case of Eigen value problem, we had no idea of what a 1 and a 3 was we expected that somehow a 1 and a 3 will materialize.

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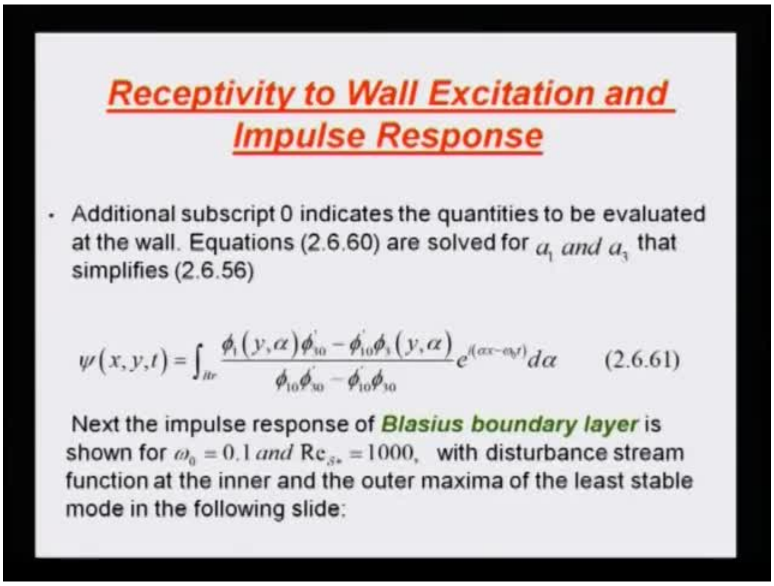

Now, if we do this as we continue solving these two equations, we get the value of a 1 and a 3, that is very easy you do that, and plug it back to your equation. Now, what I have done here instead of writing v I have written in terms of psi, is the same thing you can see the one to one correspondence between v velocity and psi component, because if I take psi like this, v will be what d psi dx, so that will be simply nothing but i alpha multiplying this phi.

So, that is why I am saying that well normal compound of velocity, and steam function has the same structure, and since we are putting in some kind of a mass source, so it is better that we work on psi, and that is what we have done, and solve for a 1 and a 3, and plug back the whole thing, and this is what you get what is a 1 is phi 3 prime 0 divided by this quantity, and what is your a 3 a 3 is minus phi 1 prime 0 divided by this quantity, and what is this quantity, it is very familiar to us, this is nothing but your characteristic determinant.

This is how we define as the dispersion relation, this is how we define as the Eigen value relation; so, that is how it appears. Now, you can very clearly see the role of dispersion relation, dispersion relation works like an amplifier, because it is in the denominator; so, in the neighborhood of an Eigen value what it does, so this quantity approaches 0. So, whatever the excitation field is that, gets multiplied by very large quantity, and that is how you should really interpret Eigen values; Eigen values are nothing but it is not black magic, it is very definitive that is why you see the connection, that when you pose it as impulse response the characteristic determinant are depression relation appears to the denominator, in this form that it appears in numerator by denominator then what happens.

Now, you see that is where your mathematics comes, in that if your denominator is 0, even if the numerator is 0, that is all of the possibility, that you will have a finite citation, this is what actually drives stability analysis, on saying that even in the absence of the disturbance, I will have a 0 by 0 form that may relate to some finite.

However, from a physical point of view as practicing engineer and scientist student, like to interpret it, let you probably would not have the numerator virtually equal to 0, there would be always some disturbances in the background, and if those disturbances also inhabit, near the dispersion relation, then of course, you will get an amplification, there is though a case by which you could switch off this quantity; suppose, I can reduce the magnitude of this quantity pi 1 prime 0 and pi 3 prime 0, then what will happen, if I keep on selectively removing them, I will not see that in response, remember the experiment of Reynolds, remember the experiment that I showed you that flow positive cylinder for Reynolds number 53 case, for different speed I have different background disturbance.

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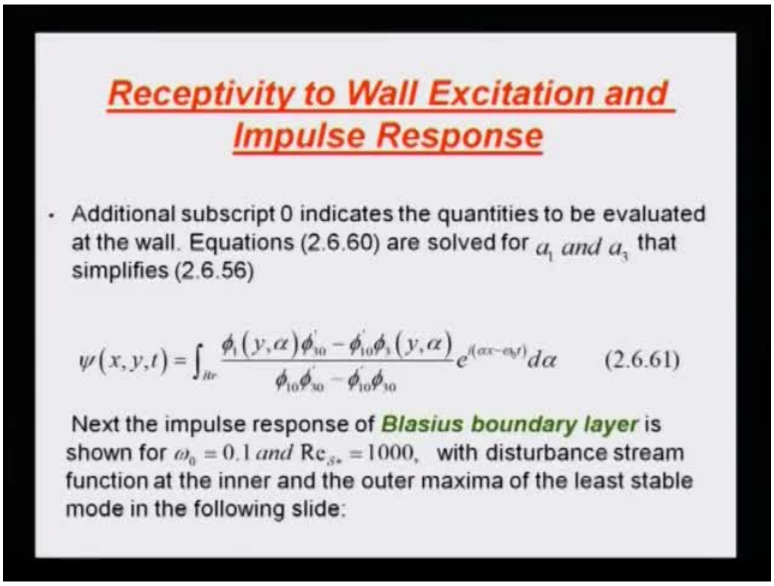

And that was your question in the mid sem Reynolds experiment relates to is it a question of nonlinearity or it is a question of receptivity to me, it appearsm it is a question of receptivity, if I can stop the background of that particular class of disturbance, where the dispersion relation resides, I would not have that response, and that is why people have gone ahead. And obtain Re critical for flow positive cylinder to a very large value, that has gone as close to as 100000, while Reynolds obtain it around 12830, what it essentially tells, that as an engineer if you have to design something, you can always design a device, where you could subdue this disturbance at that critical points, at the dispersion relation part, at the Eigen value relation part, and you can do that, and this is what you see in most of this aircraft which uses so-called natural laminar flow air falls.

There the contouring of the air falls is done in such a manner, it is a passive device, you are not exciting a ribbon or something or anything of that kind, but you contour the profile in such a way, that the pressure gradient is such the curvature is such that your background disturbance is which are there they do get less respective, because what happens by changing the pressure gradient, what I may doing, I am changing U, I am changing U double prime.

So, you can see the transfer function of the system can change. Now, you see another aspect of the designer activity that can relate to what we are discussing, there are two aspects, if I want to do something, if I decide to control some observable phenomena, then there are two ways of doing it number 1, stop the input number 2 change the transfer function, so what you see in all Boeing 757 onwards or what you see in the air bus 340, it is basically a desire to change the transfer function, because it hardly ever happens that you can change the input in a real life; of course, you can select a cursive altitude etcetera, but if you are an airline operator, you will have to be flying at considering other considerations, for example, you would be looking for fuel economy that will decide upon your flying altitude.

So, this are some of the issues that are very relevant, still not completely digested, we do not have given out all the proper way of how to handle these issues, for example, talk about this clear air turbulence, there is a case by which we have figured out, that if you have a vortex atmospheric convicted vortex is approaching the aircraft wing, then a pilot can decide whether to go over it or below, it provided he knows the sign of the vortex. If you are talking about a vortex which is counter clock wise rotating, it would be very advisable for the pilot to duck under it, and then instead of destabilizing the flow, it will stabilize the flow, we will come to that discussion in the next topic.

But what I am saying is now, let us not jump the gun, we have two quantities at our disposal, one is controlling the input, the other is controlling the transfer function. If you are a designer in the shear flow, you would rather change the transfer function, you do not do get to change the inputs, whereas if you are an experimentalist, you are interested in understanding the flow physics, then you do play around with the input itself; so, these are the two different complimentary activities.

So, now what we are going to show you, the very clear cut case, we take a Blasius boundary layer, and we take Reynolds number of 1000, that would perhaps contribute to somewhere here. So, where we have some kind of a power of boundary layer, this delta star relates to the displacement thickness, and we have already seen, that Reynolds number of 520 is about the point, where flow becomes first critical. So, 1000 is faring to the super critical range, and we are talking about omega not equal to 0.1, that would sits somewhere in the middle of those neutral curve; so, it will be quite perceptible disturbances that you are going to see.

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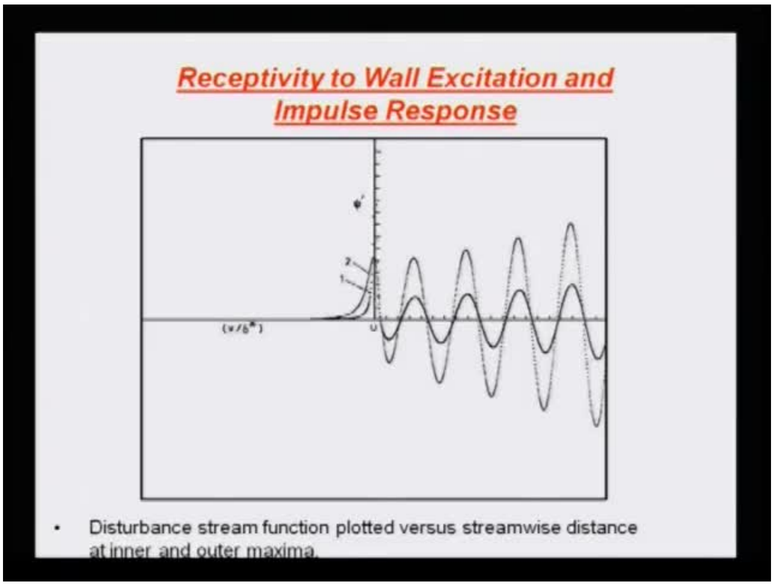

Now, so move ahead and see what it looks like they looks like, what we also get a qualitative picture from stability theory, that we see the disturbances plotted at two heights; the one the solid line, it is the outer part of the shear layer, and the larger amplitude is the one, that is closer to the wall, so that is called why inner maximum.

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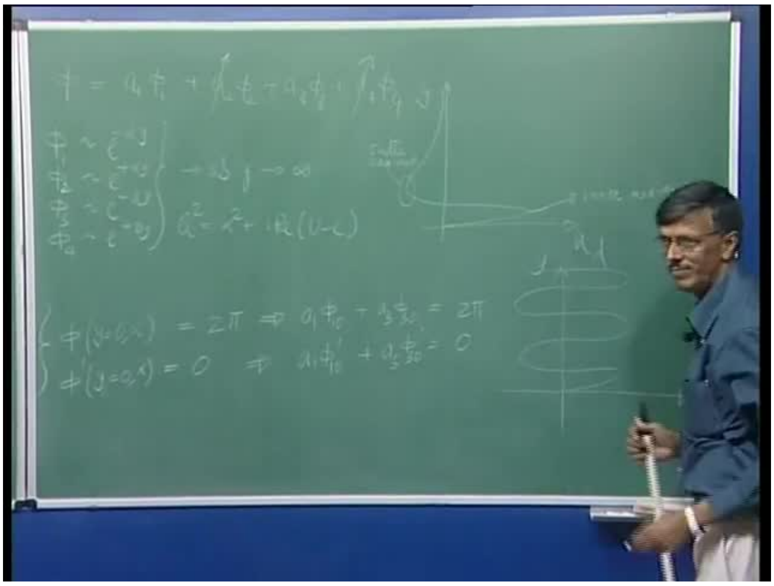

So, you will see that if I plot the disturbance velocity versus y, the disturbance velocity looks like this, and this is what is called as the inner maximum, and this part is called the outer maximum.

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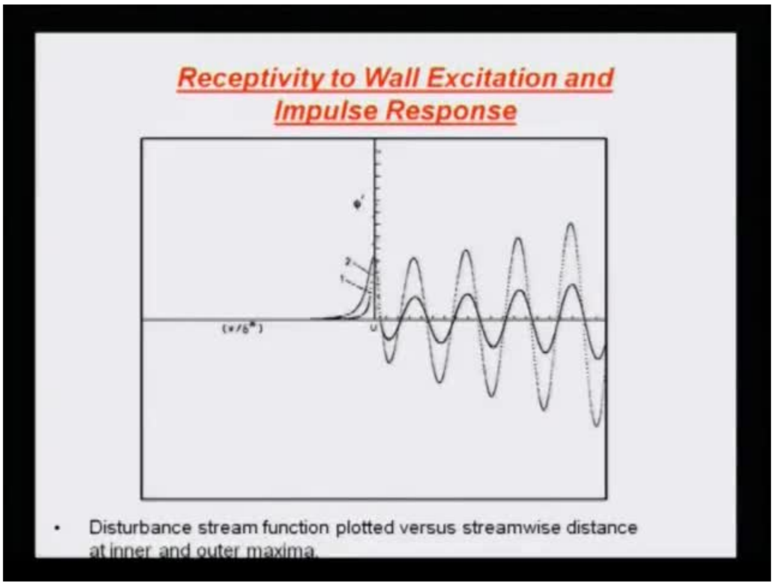

So, basically talking about the magnitude, the disturbance quantities have 2 maxima, one here, one there, if I have to ask you which one would you prefer, of course, you would point out this one, because this is a larger signal, that you would get this magnitude is much larger compare to this, and that is what you are seeing here the inner maximum has a larger amplitude compared to the outer maximum.

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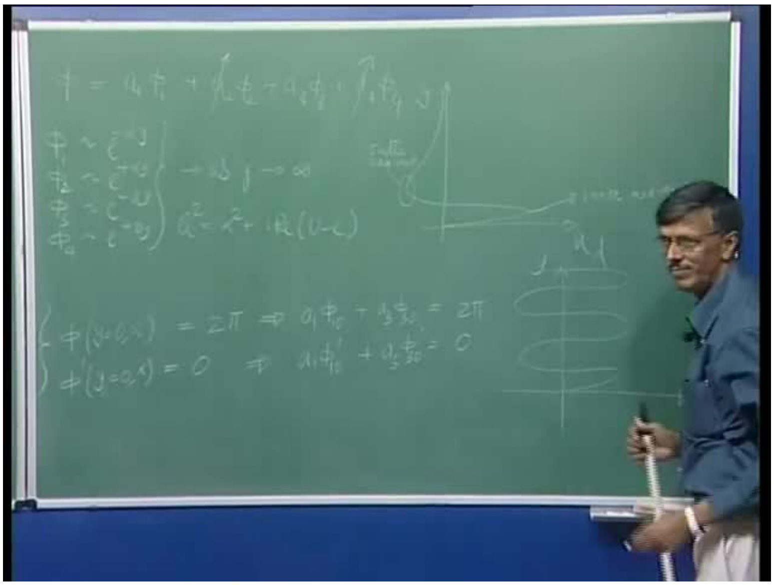

But what happens is this is a very narrow, and if you are trying to do a measurement, if you are an experimentalist, it is rather difficult, if you miss it, if you are not there, if you are slightly ((poor audio quality)) will be there; so, tracking the inner maxima exactly is not a very straight forward issue.

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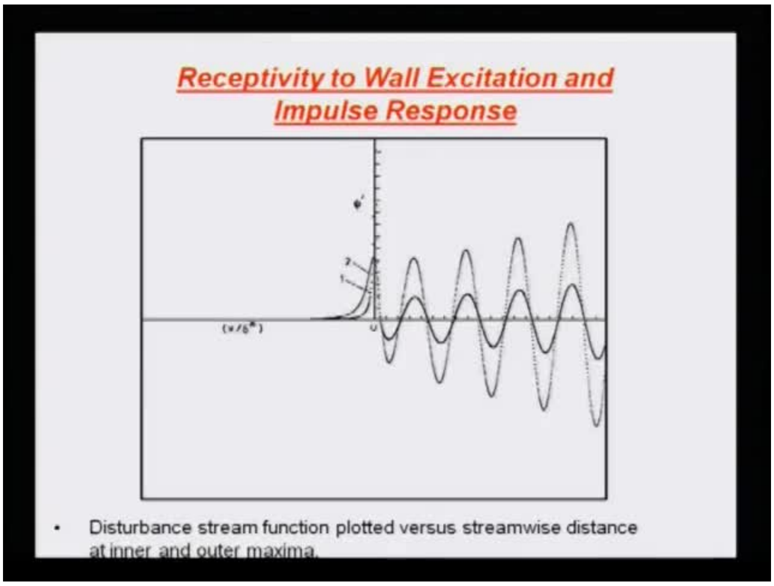

So, experimentalist like to focus on the outer maximum, so that was one of the reason that we plotted this, and you can see what the linear theory did talk about the amplitude decades exponentially, like what we wrote here e to the power i alpha x minus omega naught t.

So, with x the amplitude is growing exponentially, however, of course, your stability theory would not be able to tell you this part of the solution, this is what we call as the near field of the solution, because where is your exciter, exciter is at origin, exciter is at origin, so the respective theory actually tells you that what happens in its immediate neighborhood, and what happens asymptotically fall down.

So, stability theory basically tells us what happens far away from the exciter; so, this is something we need to understand, I will talk about a couple of theorems attributed to (( )), and tower shortly, and then, you will see that, this is what we would expect, because we have talked about the complimentary property of Fourier transform, and the direct inverse, and the inverse transform and the direct transform, what did we see, that if a quantity is localized in space, that in the case space, it is in sort of all (( )) and vice versa.

Now, if I want to see something, which is far away from the source of excitation, then what should I have, that point would be where, that should be in the unstable part, and for x positive what should be the nature alpha, alpha i should be negative; so, what happens is, alpha i negative values, if they are there, then you are going to see it everywhere all its, but what happens in a realistic flow, like what we are studying here even a Blasius boundary layer, this points are very close to the origin.

So, what happens that you going to see a kind of a moderate growth. So, we will again come back and talk about that couple of theorems, and will find out, what happens is that, if we try to talk about this, in the neighborhood of the exciter, where should we look at in the alpha plane; this we should look at far away from the origin, because it is the complimentary problem.

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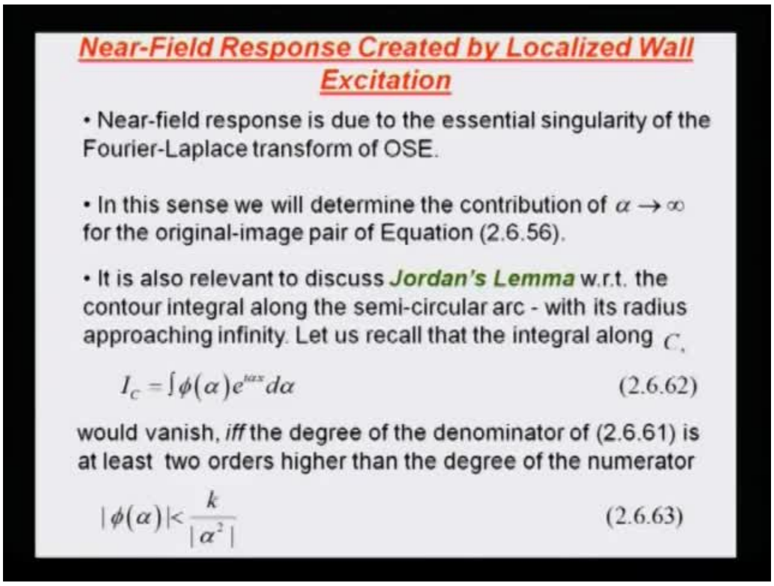

If I want to see at x equal to 0, I should look at alpha equal to infinity, and if I want to look at x equal to infinity, I should do it very clearly near the origin that, this is the complimentary property that we talk about. So, this is one of the finest achievement of the receptivity theory, that we get a composite picture, in fact, not many people before 80s have the good appreciation of this thing what we just now saw.

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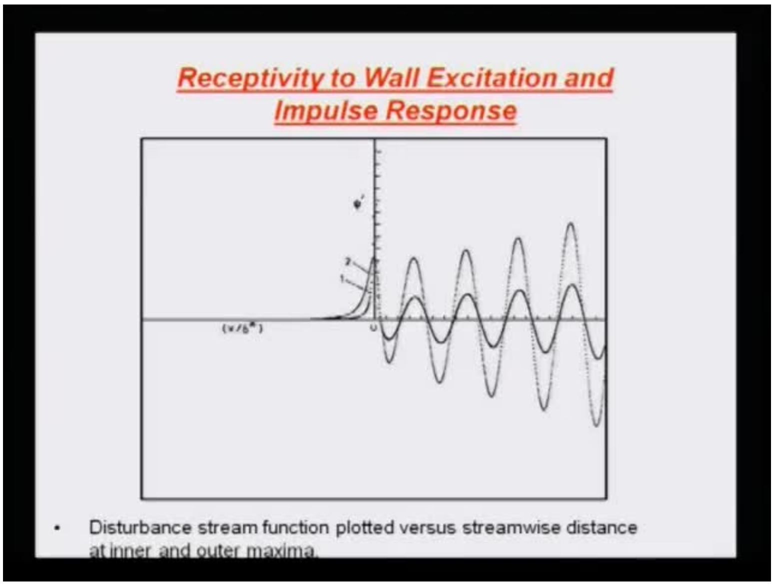

The disturbances do not appear in a discontinues manner, you had to switch back to the previous one, we can see that despite the fact, that the exciter is placed here disturbance, thus penetrate little bit upstream, how far this is this not very much, this is about 30 to 50 delta star. And the penetration is the restricted, but it still in a continues manner, if you look at any book except this one, that we are following here, you will find that there is a simply do not there say, what is here, they will they simply say something happens there, and then, we get this, but I must also tell you, that if you look at even in the positive x side, immediately on this side of the exciter, the property is do not correspond to what you see in there, why, because that is what I told you, then the neighborhood a x equal to 0, the contribution does not only come from the Eigen values, it thus comes from alpha going to infinity also that is what we would expect.

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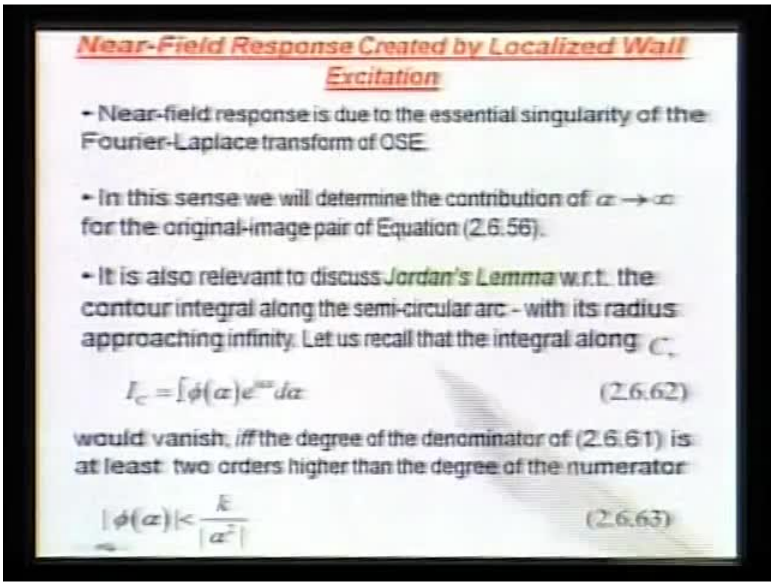

So, to understand that the near field has this particular feature, which do not reflect what your Eigen value analysis tells you, and stability theory would never tell you that part now I already said it, that the near field response that we are see is due to alpha going to infinity point, and what that is called, that is called the essential similarity, that you have done in your complex analysis course, like if I talk about e to the power z or sin z or cos z, they are not define, when z goes to infinity those points, I call the essential singularity. So, here also in the alpha plane alpha going to infinity would be the essential singularity and I explain to you why it should be so, because they will contribute significantly to your near field response. So, there is a case for obtaining the contribution from the essential singularity alpha equal to it, that we should try to plug it in this kind of relation, this is your physical variable and this is transform.

So, that is what we say the original image pair we should know. Now, if you recall when we were discussing about the causal integral, and the property is of the Eigen values, what did we see we use that Cauchy’s integral formula, and we indented the contour along the Eigen values and closed it.

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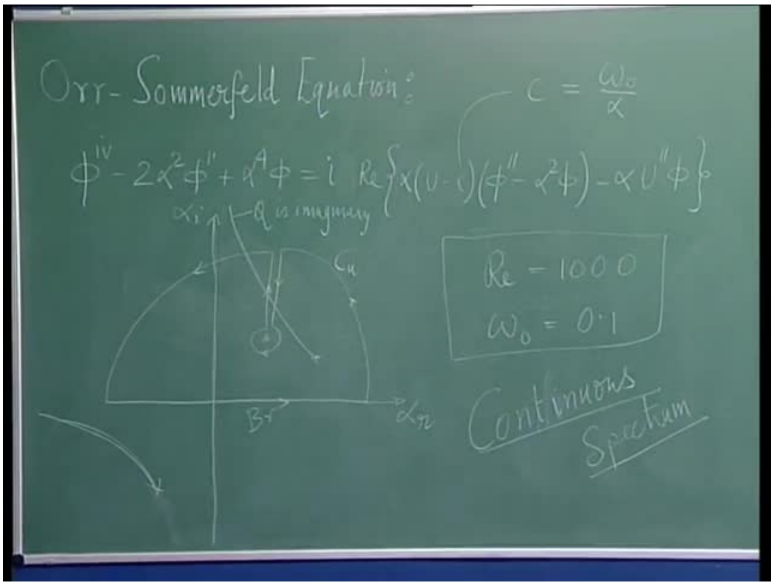

However, what you also did, suppose when Eigen value here, then what I say that, I go along the Bromwich contour, and let say the Bromwich contour is along this axis, and then, I decide to close it from this side, and then, what I said I must bypass, and close the contour like this.

Remember this is what we are done, so we are showing the connection between receptivity theory and instability theory time, and again, and this is the way, we go about doing it. We exclude the singularities, so that the function is analytic everywhere, and then, we say by Cauchy’s integral formula which should be equal to 0, it should be equal to 0, it is fine, but then, it has the couple of contributions one comes in the Bromwich contour, other comes from the contour which is in the upper plane.

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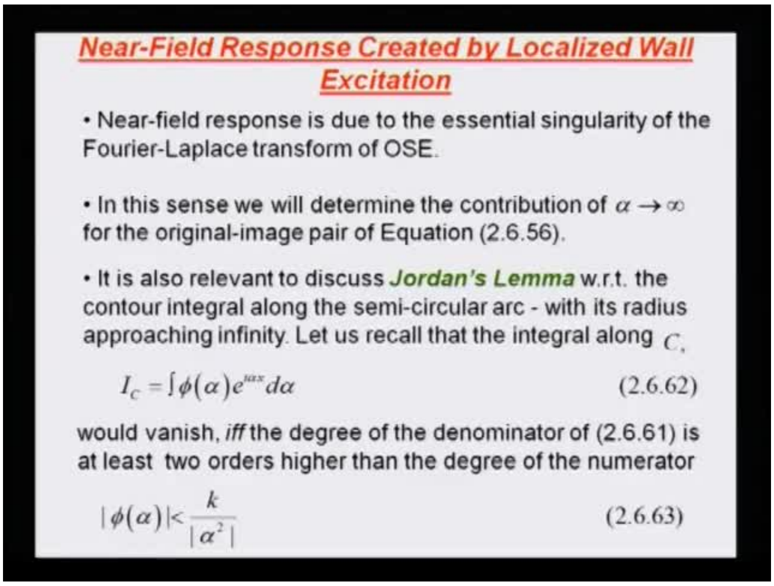

Now, when we say that the contribution coming from Bromwich contour is equal to some of the contributions comings from the residue at the poles, that is where, we assume that the contribution along the semicircular arc is 0, and when we do that, that is what we call as the Jordan's Lemma, the jokingly at we call it Jordan's Lemma, because it so happens, that in this particular case, we will see that it is not always given that, this quantity will be equal to 0, unfortunately in your problem math's scores, you have 100, very nice, well we have functions which satisfy Jordan's lemma, nobody has shown it so far, that Jordan's Lemma holds, even for this Orr-Somerfield of equation, and we did try doing that, but what Jordan's Lemma stands for is this observation, that if I am performing a contour integral of this kind, then this phi of alpha should have this kind of a structure.

It should have a again a numerator by denominator, like what we saw in the morning to day itself, however, it should not be in a rational fraction kind of thing, but the order of the denominator mask be at least 2 degree higher than the order of the numerator.

If it is not, then you cannot show that Jordan's Lemma is valid. So, the task is very simple for us ask to investigate, we need to figure out for our case, the solution of Orr-Somerfield equation phi of alpha, as alpha goes to infinity, what is the order of the numerator and denominator, it is very interesting that about more than 20 years ago, I did the sit down and worked out.

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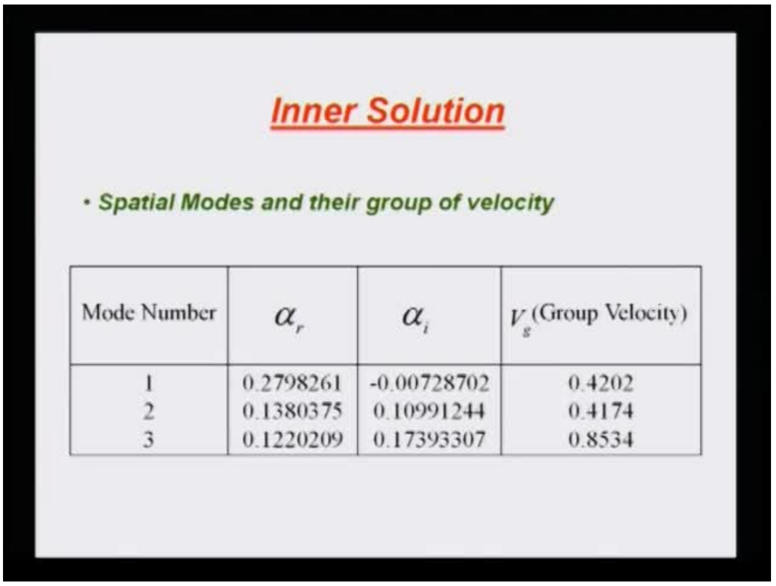

There is another philosophical issue, that often remains, and said in most of the stability approach is that, if I had to the only depending upon the Eigen values, then when, and how I can express the disturbance field in terms of this Eigen values.

If they are finite in number, it is virtually impossible to describe any arbitrary function, my disturbance environment could be anything arbitrary, and if I have only finite number of Eigen values, then I have problem. So, people have try to address this issue, they try to say, something somewhere must give rise to I helping, and so that even with the finite number of Eigen values, we can describe any arbitrary disturbances.

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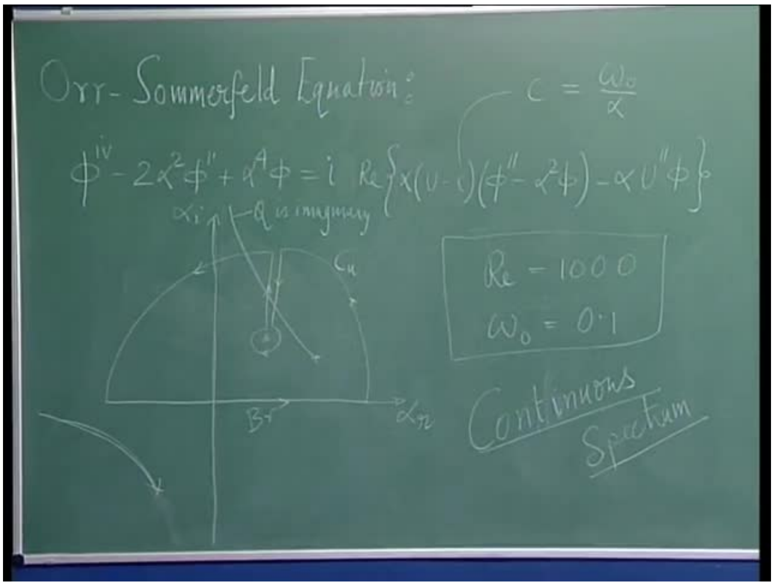

Now, you look at the simple case what we were looking at, we are looking at the receptivity of the Blasius is boundary layer for Reynolds number of 1000, and omega not equal to 0.1, and we now know how to find out all the Eigen values in a finite part of the domain by that grid search method we have studied.

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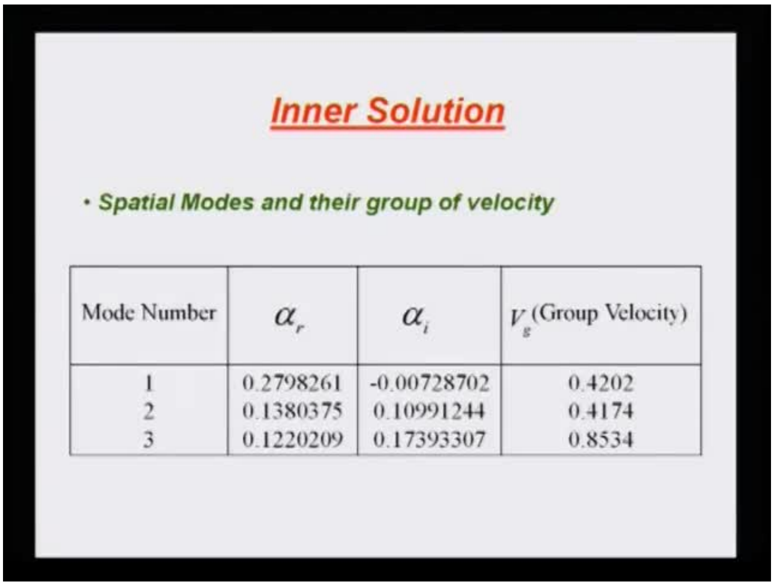

Mode Number	α_r	α_i	V_g (Group Velocity)
1	0.2798261	-0.00728702	0.4202
2	0.1380375	0.10991244	0.4174
3	0.1220209	0.17393307	0.8534

So, if I do the grid search method for this parameter combination, no one behold you only have three modes, and those three modes are, of course, ordered in this particular manner in the way the magnitude of alpha i increases, what is the first one, first one is unstable with the minus sign this is, what is called as a Tollmien-Schlichting mode or Tollmien-Schlichting waves, this is what Tollmien-Schlichting first found out, and your Eigen value analysis tries to obtain this Eigen value analysis can also hope to find others, but to my utter amazement about more than two decades ago, I found these are not very well done, and we develop the compound matrix method, and we used the grid such method, and we figure out in a box in alpha plane, say of size 2 by 2, and all side, and we only figure this three Eigen values. These Eigen values is loss 2, as you can see comes under plus sign, so if they were to be going down stream, they are stable nodes, how do you find out which way they are going calculate the group velocity.

And we calculate the group velocities, all of them are positives, so they are all downstream propagating waves, and then this is an unstable wave, and this two are stable waves, now my question that I asked you about that with only three Eigen values, can I hope to represent any arbitrary case, you have already seen with the help of Fourier series and Fourier transform, we try to express any arbitrary function in terms of infinite number of terms, a finite number of terms will not allow us.

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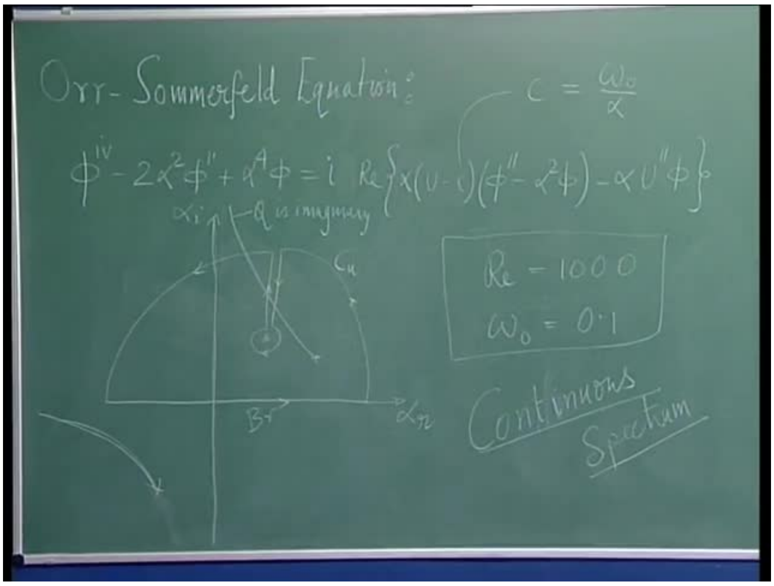

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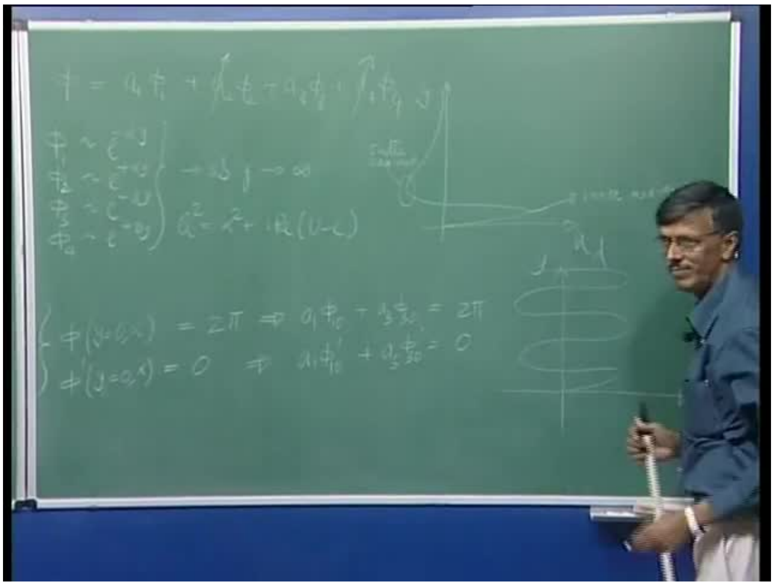

There are lot of activities at the fluid mechanics, and physics community in trying to say what it is. Now, you see what it is people, of course, duct upon the wrong tree, they did not look at the Jordan’s Lemma part, instead they said somewhere, from somewhere else something is coming, and this somewhere or something, they attributed to as a continuous spectrum, where would this continuous spectrum be, well for example, if I am plotting in alpha r alpha i plane, you see all these concepts holds good, if we are not along the imaginary axis.

If we had a long the imaginary axis, then can we say 2 and 4 going to 0, we cannot say that, so once group of people did say look maybe along this axis, we have a contribution coming from all the points, and this is what they called as the continuous spectrum. Now, that is about a 1, a 2 contribution, what about a 3, a 4 contribution, that depends on q; q also can be purely imaginary your expression is given, you can work it out, all give it you as an assignment work it out.

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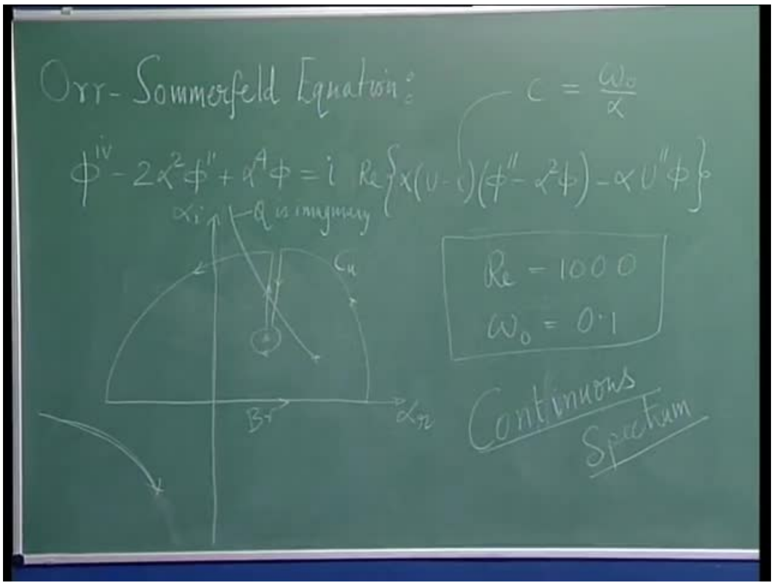

And find out the curve or curves along which Q is purely imaginary, and if you have such a curve, which you would be able to see, there would be two such curve along which Q is imaginary, one such line will be there well originating from some point, where Q will be equal to exactly equal to 0, and then, you will see it will go along this, and then, you will have, yes, similarly another point would be somewhere here, and then that would go like this.

So, you can work out as I told you, that this might be one such special line, and there could be this couple of more lines on their, along which some of these fundamental modes could be purely imaginary, and then we can see, but then are they singularities this alpha i axis or this two lines, that I drew, so I mean, there is no reason, why it should be, and we have already talked about while deriving the initial conditions for solving the compound matrix method, how to handle this particular transition from this side to that side, we just simply switch of this.

So, is that equivalent to basically going along this and coming back, and doing this is that, what it is that, if I go all the way up to close to the origin exclude the origin, and then, come back to the other branch, so is that equivalent to doing this if that so, then we cannot even have a close contour, because this alpha i axis, and semicircular arc will cross each other, I mean, do it is a tough mathematical conundrum, because we are talking about point at infinity, going along this line or going along by changing the radian.

So, it is quite unlikely that is should come out, I would advise you to think, and analyze yourself, and see if a contribution from here is going to be generic for any arbitrary type of disturbances.

If you have let say purely imaginary exponent in phi 1 and phi 3, what kind of disturbance structure we are talking about, with why a never dyeing disturbances all the way have to infinity, but we get such disturbances I mean think of it.

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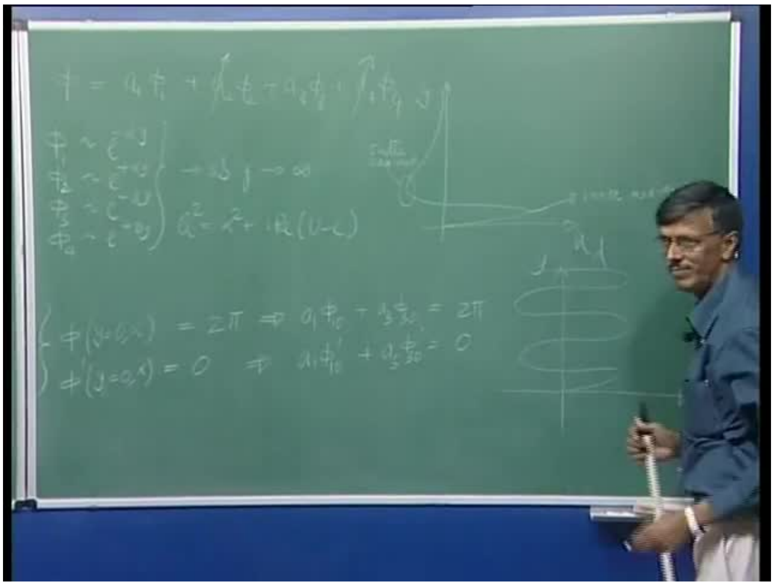

Such a disturbance will have infinite energy, it is quite impractical, but still some physicists some mathematicians have spent some time, and there are the camp follows, which I do not subscribe to, for me it appears that is a very pedagogic exercise, does not hold water from particle aspect, because we cannot have seen here, I shown you five structure is like this, it remains very much buried, inside the shear layer as you go out it decays.

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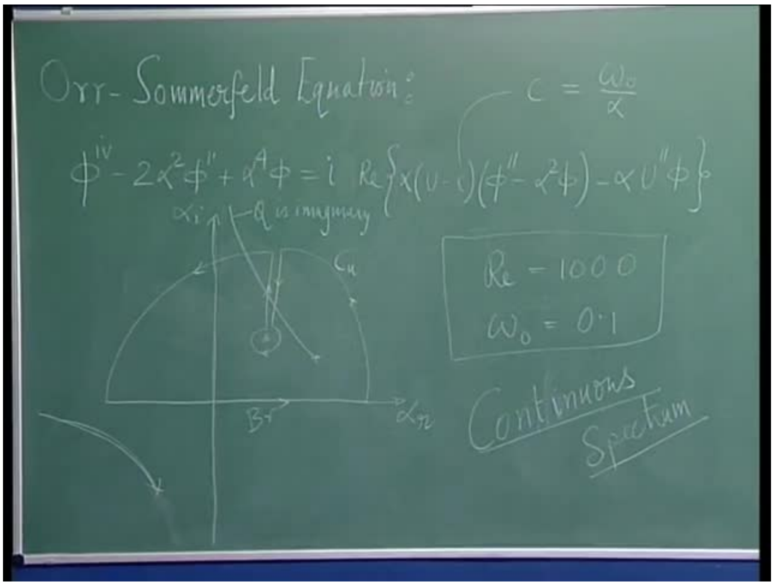

But along the imaginary axis, I would have ((poor audio quality)) plot phi versus y, basically we are talking about some disturbances which will go like this forever, it is quite unthinkable (( )), a 1 and a 3, a 1 and a 2 are also function of y, but that is not a very good thing to assume (( )), because that a 1 and a 3 can become function y, for some other reason not for linearized Navier Stoke equation with parallel flow approximation.

We will see that later, that when we do nonparallel theory by solving Navier stokes equation, we would see that there is an additional dependence with y, but that is not apprehend directly from Orr-Somerfield equation, I know these are not so easy concept, but we need to get used to the idea of analyzing, and saying what is feasible, and what is not, to my mind it appears that continuous spectrum at along the imaginary axis or along q r equal to 0 line, that is all we are going to generate, along this two line q r will be equal to 0.

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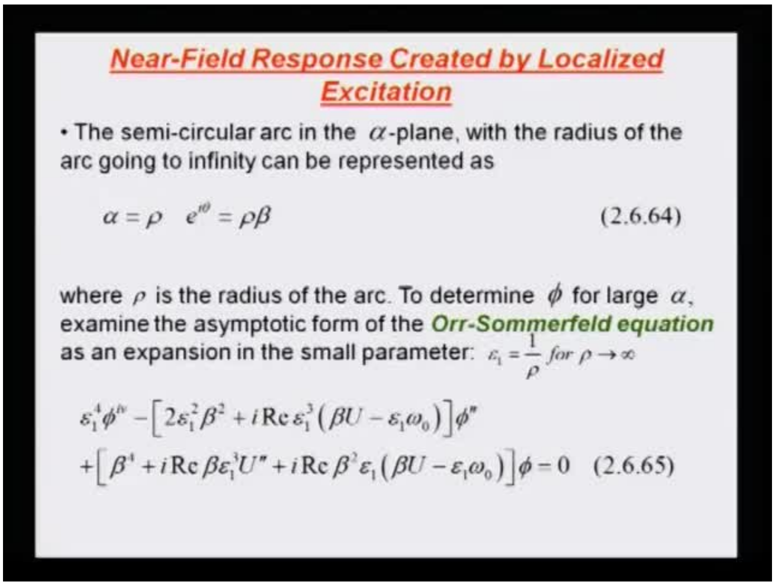

So, you can work out the value of q r, and you can come out, and tell me what this is, and maybe, you should give it as a summation, and tell me what are these two lines. Now, continuous spectrum may not do the trick, but we have already seen that there is a possibility that Jordan’s Lemma can be evaluated for Orr-Somerfield equation, and that might contribute something, and this is something we did, and let me show you what we did this is quite simple, but it was not done before, so let me explain to you how we go about doing, what you do is basically you are trying to figure out what is the contribution coming from that semicircular arc.

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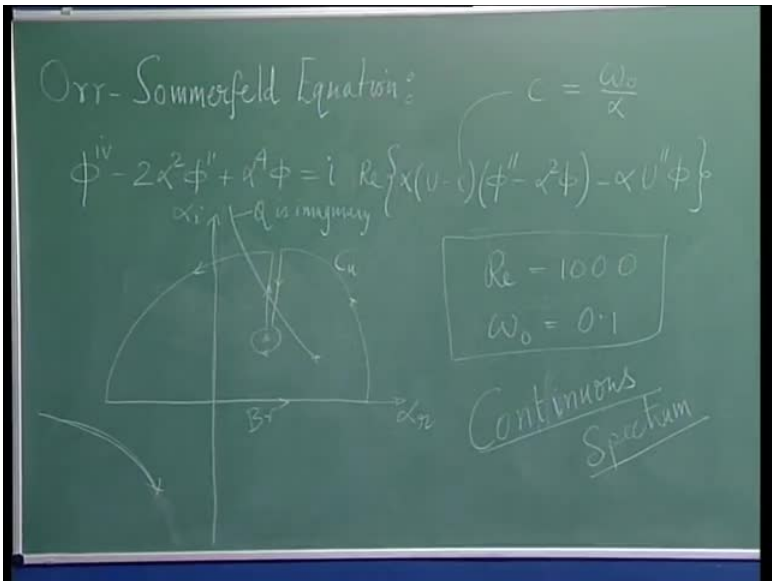

If I then write alpha is equal to some in a polar representation the radial vector rho times e to the power i theta. So, for simplicity I just simply write e to the power i theta as some beta, so rho is a real quantity beta is a complex quantity tells you about the phase cos theta plus i sin theta, you also realize very clearly that this semicircular arc is what this is alpha equal to infinity, when rho goes to infinity; so, it is interesting that in a polar representation, the point at infinity is the whole circle, when the radius goes to infinity.

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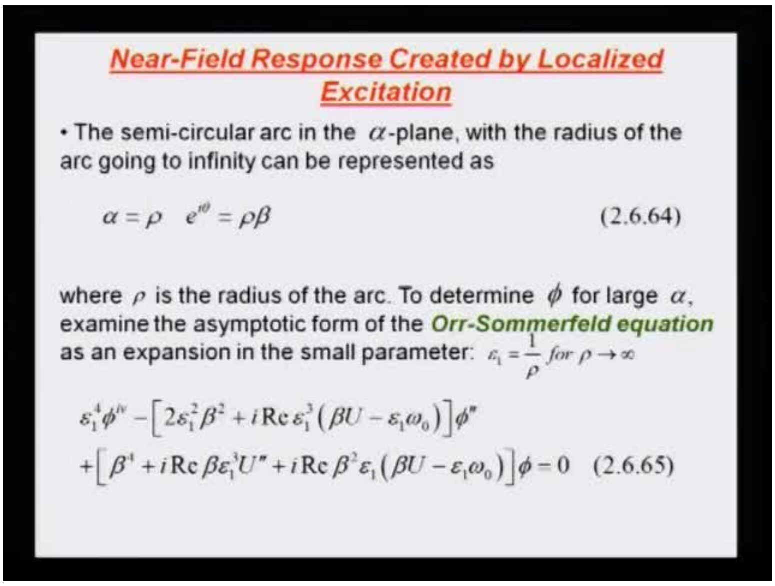

So, you got to remember that is what we are doing, where we close the contour from the upper part, we are looking at half the contribution coming from infinity and other half would come from the bottom, so we are looking at it that way, so let us see what happens, we are now trying to figure out, what this phi is going to be for alpha going to infinity.

So, that means rho should go to infinity, so what happens I can define a small parameter call it epsilon 1 which is 1 over rho, as rho goes to infinity epsilon 1 goes to 0, and then I have the Orr-Somerfield equation, and then, what I would do is, wherever alpha appears, I will write it in terms of what beta by epsilon 1 is not it, beta by epsilon 1 and I plug it in, and expand it, this is what I get, so basically we are writing the Orr-Somerfield equation further limit alpha going to infinity, in terms of a small parameter epsilon 1, very simply done, there is no trick involved here just to see that.

However, when we write this equation down, this equation 65, we are troubled by one aspect that, the here the highest derivative term is 5 4, and this is multiplied by the small parameter, so what do we get; we get a singular perturbation problem, you see perturbation theory as the subject was developed following Prandtl's work on boundary layer theory, has two branches one is called the regular perturbation theory, another is called the singular perturbation theory.

Regular perturbation theory be regularly, so where they are no issues you can expand the problem in terms of a small parameter, and you can slowly obtain those one term after the other in the perturbation series, that is your regular perturbation theory.

In a singular perturbation theory, like the boundary layer theory, it is all phase the highest derivative term is multiplied by the small parameter, and if we try to go through the regular perturbation theory analysis, then we see that we are not able to satisfy all the necessary boundary conditions, that is one of the problem with singular perturbation theory problem, because you cannot just simply go to eliminate epsilon 1 going to 0, if you do, then immediately the highest derivative term disappears, and then, you will not be able to satisfy all these four boundary conditions that we have been solving so far.

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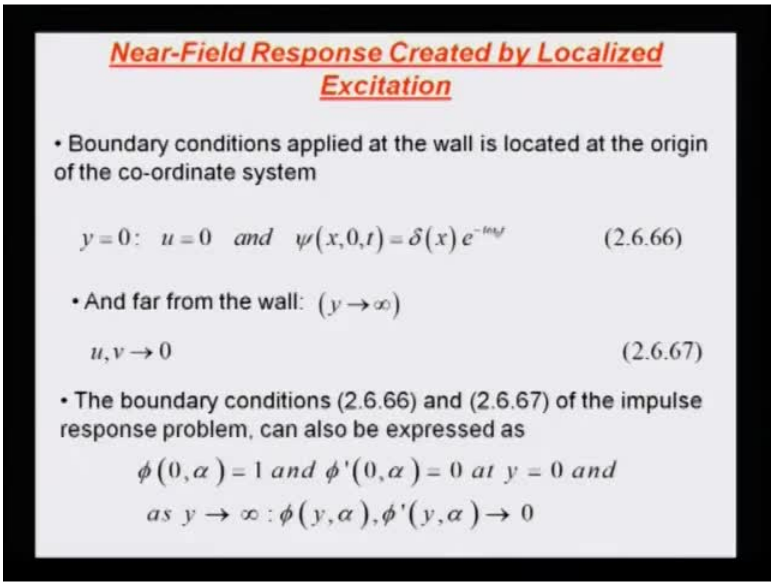

So, what happens is singular perturbation theory has to be dealt with in a somewhat different a manner, I would not go into the mathematics of it is pretty straightforward, if I look at the book by Bender and Orszag, there will be a very decent discussion, but let us see how we can go ahead and do this.

So, if we are now trying to this, we are trying to find out the contribution coming for the disturbance field from phi, when alpha goes to infinity, that would give me the near filed response; so, there is the reason, that we are calling this contribution as near field response, because we are trying to compute, what is the contribution coming from alpha going to infinity, well our original problem remains as it is.

So, we have at the wall u equal to 0, and psi equal to this, and of course, far from the wall, if we go outside the shear layer u and v equals to 0, well these are the physical plane description the corresponding spectral plane description is this right.

We have seen that this comes out like this, so I have been sloppy somewhere I write 2 phi here, I kept 2 phi 1 it does not matter, you can scale it up; so this comes from this condition, from this condition we get this, and these two conditions are written like this, and lets know trying to solve that singular perturbation problem with this four boundary conditions without giving up any one of them.

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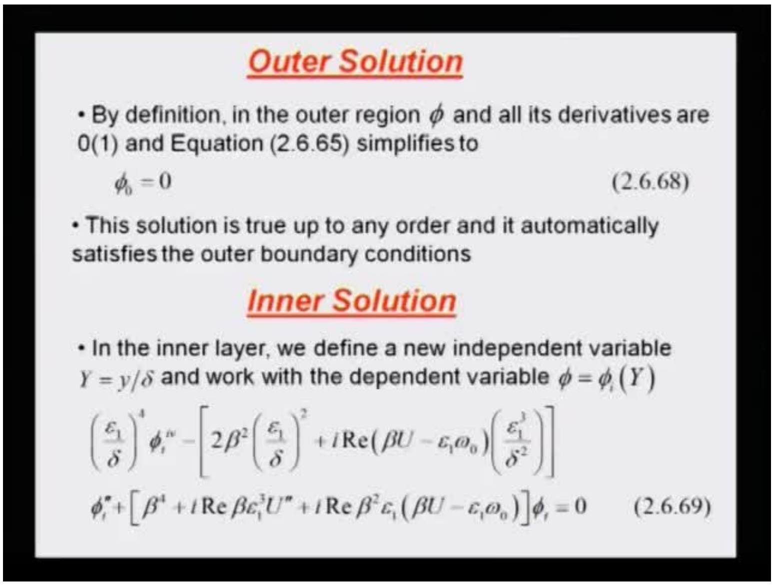

Now, when you look at the singular perturbation solution of an equation, like so what we have written, you get two parts of the solution, like what you have seen in the boundary layer, boundary layer flow is what a two deck structure, one is the inner deck, which is the boundary layer, and there is the outer deck or which is the potential flow.

So, in the potential flow what you do, you allow Re to go to 0, that is what all I did, that is what Laplace did, then what you get, you get rather the Euler equation or you get the Laplace's equation, that is that solution is independent of Re; so, same thing we should do here, we will talk about outer region, so it is very interesting, now sum it up for me, what we all looking at, we have a real flow which has a singular structure in terms of a boundary layer on a potential flow.

Then, within this boundary layer what have we done, we have created some disturbance, and within this boundary layer now we are talking about another similar perturbation problem; so, within this shear layer, I have an inner layer and an outer layer.

So, this outer layer within the boundary layer should have the property, you allow epsilon 1 over to 0, and if you do that in that equation, we have seen just now, I will make this transforms is available to you, so you can take a look for all this solution that survives is phi naught equal to 0.

So, it is a very clear cut something that, if you all looking at the boundary layer of the Orr-Somerfield equation, please do understand those boundary layer is within the quotes, this is not the physical boundary layer that we have investigated so far.

So, this is a boundary layer of the Orr-Somerfield equation, and outside that boundary layer the only solution that is feasible is phi naught equal to 0, there is no problem with that, we can always have a case where in the outer part.

We would not have any contribution, and this is interesting, this is true for any order of epsilon naught and what happens this satisfies this far field condition, while going to infinity, there is absolutely no problem to any order you can satisfy that.

What do you do, when you try to obtain the boundary layer of the Orr-Somerfield equation or the inner solution of the Orr-Somerfield equation in the limit alpha going to infinity, what did you do with boundary layer itself. We actually stretch the wall normal coordinate, that is why we got this similarity coordinates, if you recall from your first fluid mechanics course, when were told about boundary layer, how do you get the boundary layer.

You stretch the wall normal coordinate, so same thing we need to do, so we need to define a new independent variable, which we will call it as capital y; capital y will be divided by the boundary layer thickness of the Orr-Somerfield equation. So, this delta has got nothing to do with the delta of the boundary layer, physical boundary layer this is a mathematical boundary layer of Orr-Somerfield equation; so, we introduce this, then you know all those terms that we have here.

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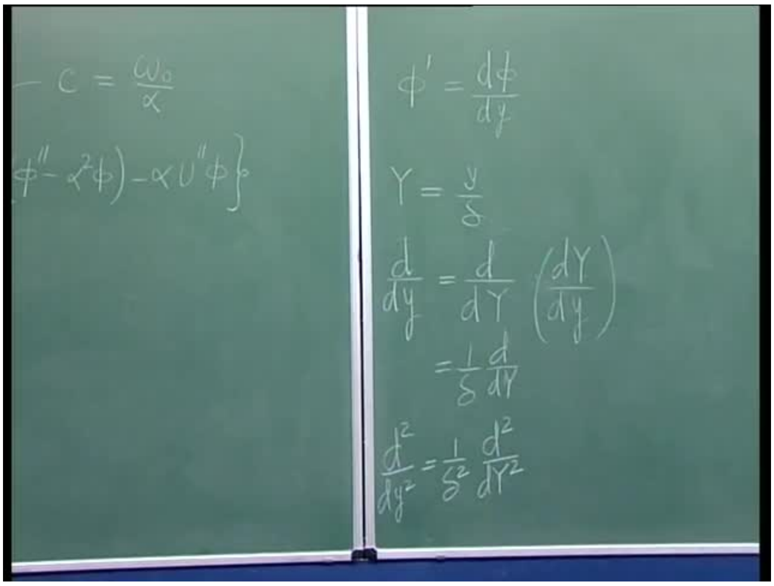

Let say for example, when I write phi prime, what I mean, I mean this, d phi dy, the physical variable. Now, what I want to do is, I want to introduce a new independent variable this is y over delta, so what I could do is basically I could write a d dy should be equal to d d of capital Y times, this very easy and what is this.

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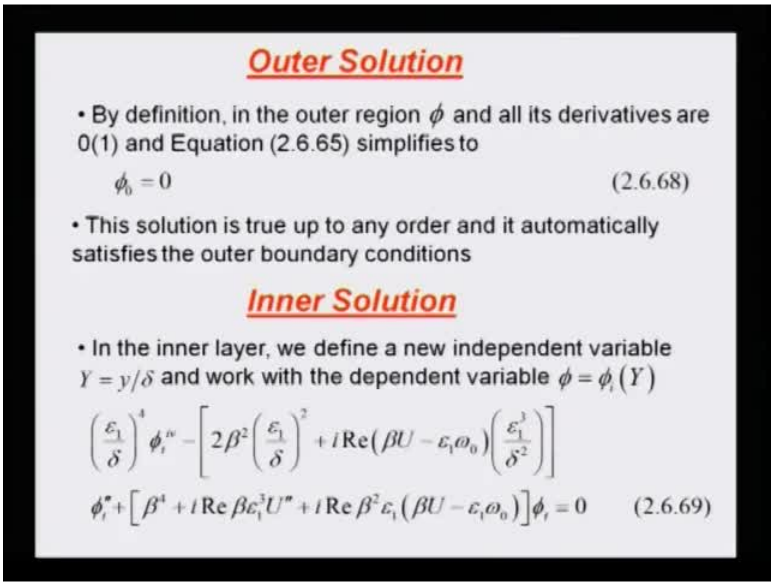

So, you can carry on the story, further you can obtain the second derivative, what will that be 1 over delta square so and so forth, and you do that, there you see what happen earlier we had a term epsilon 1 h to the power 4 and the fourth derivative of phi.

Now, we are writing the same thing, but now this prime refer derivative with respect to capital Y. so, we get this equation, now you see that was the way, we handle the physical boundary layer, we are coming around the same root. So, what we are seeing here is this equation, we will start from here in the next class.