Instability and Transition of Fluid Flows

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Lecture No. # 14

We will resume our discussion on Laplace transform and how it will be made useful for receptivity studies.

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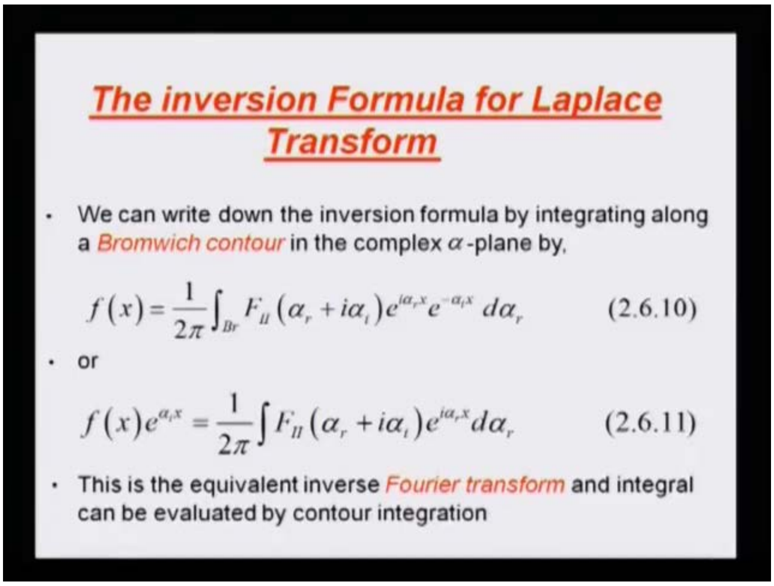

The central point is about using Laplace transform, where we have indicated how the transform relates with the original; the function in the physical plane – we will call it original. This is the transform. And, alpha is complex. So, it has two parts. This part – actually, if I take a contour along which this is a constant, I can take this out and then I can put it on the other side, then this will be this. So, that is what we are talking about that, if we follow a contour, which we call as a Bromwich contour, for which say alpha is constant, then this is the formula.

Now, this is nothing but (Refer Slide Time: 01:16) the usual Fourier transform, except that this argument is complex; unlike in Fourier transform, the argument is the real frequency, circular frequency; here it is a complex frequency. So, we discussed how we could use this.

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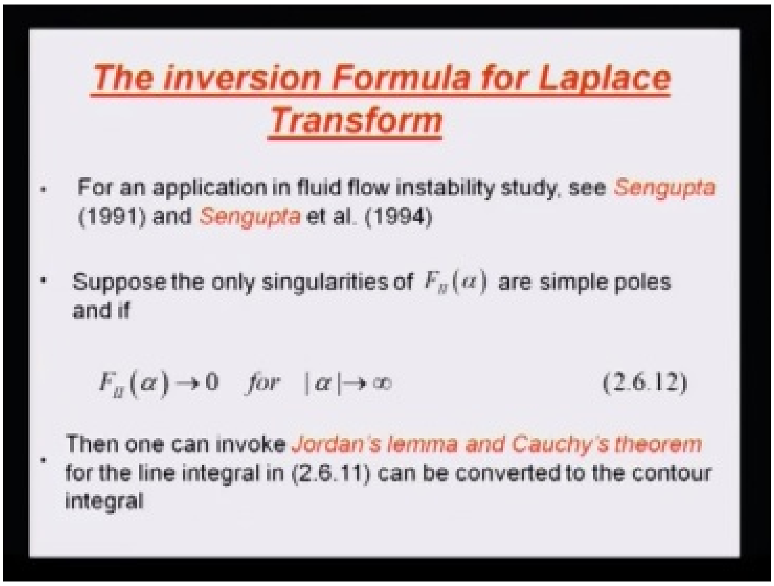

If the transform has the following property that it has only simple poles and this quantity goes to 0; the transform goes to 0 for alpha going infinity, then we can use the Jordan's lemma. And, as I said, the Bromwich contour is something like this – constant alpha i.

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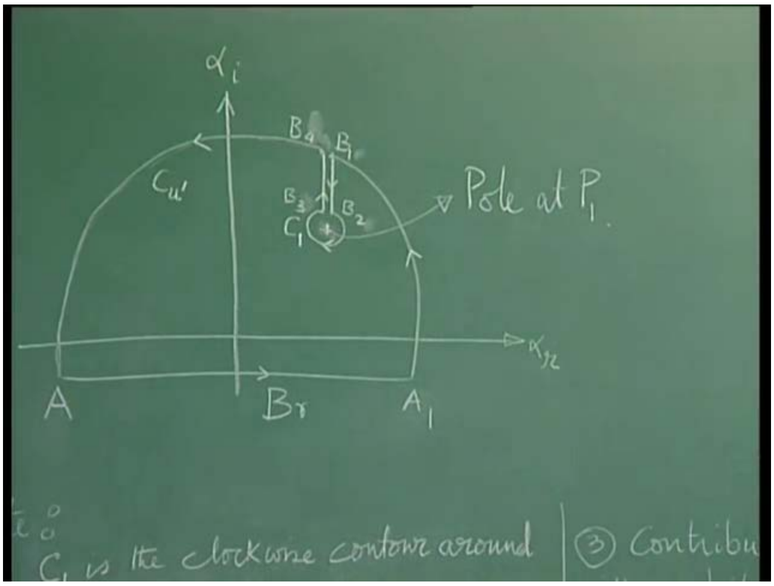

Now, what I could do is, I could construct closed contours, where the simple poles are excluded. The way we have designed the contour in a very specific way, the pole at P 1 is circumvented by indenting a contour around, which is joined with the main contour with this vertical line B 1 B 2 at B 3 B 4. So, what we are saying then that if we go to alpha equal to infinity, what does this mean? (Refer Slide Time: 02:42) Alpha equal to infinity – alpha equal to infinity is a complex quantity of alpha. So, its modulus going to infinity means, what? It is like radial vector with the radius going to infinity. So, alpha going to infinity is actually the point at infinity and that is nothing but a circle. So, what you are noticing here is half the circle. And, this semi-circle is in the upper side. That is why, I have given it some kind of a name as C u prime.

What is this? C 1 is the contour. And, you see, how we have progressed in performing this contour integral. We have gone from minus infinity to plus infinity. Then, we followed along; then, we came down; went around this way; and then, went up; and then, closed the contour. So, there are certain things that you notice the main contour is counter clockwise, but the C 1 is clockwise. You must remember that. So, contour integral depends on the direction in which you do go.

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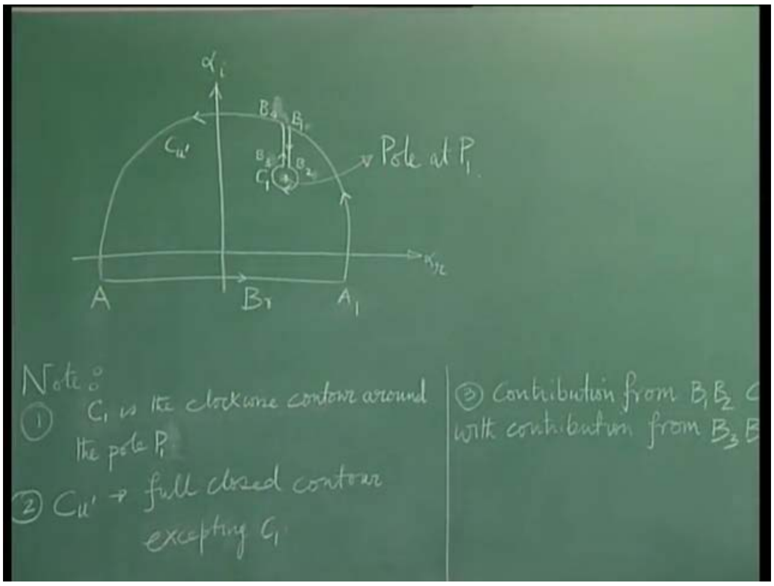

C u prime – what we are calling here is nothing but the whole contour minus C 1. That is the one thing that we understand here. The other thing is also you notice that this is kind of a mathematical indenting. So, the contribution from B 1 and B 2 will cancel from B 3

B 4, because B 1 B 2 you are going down; B 3 B 4 you are going up. So, they will cancel each other. So, basically then, the whole contour would consist of the contribution coming from C u prime and C 1. This is how we do it.

And, we are saying one more thing is that contribution coming from this semicircle would go to 0. That is what is called the Jordan's lemma (Refer Slide Time: 04:52). And, the Jordan's lemma is valid only when the transform is equal to 0. And, if a complex function has this property, then Jordan's lemma can be used; and, Cauchy's theorem states what? For analytic function, if I take a closed contour, f z dz is 0. So, that is what we have done by sort of designing the contour in such a way that this region (Refer Slide Time: 05:24) – everywhere the function is regular, analytic; only pole is here and that has been excluded from the region. So, f z dz over this whole contour should be equal to 0. That is the Cauchy's theorem.

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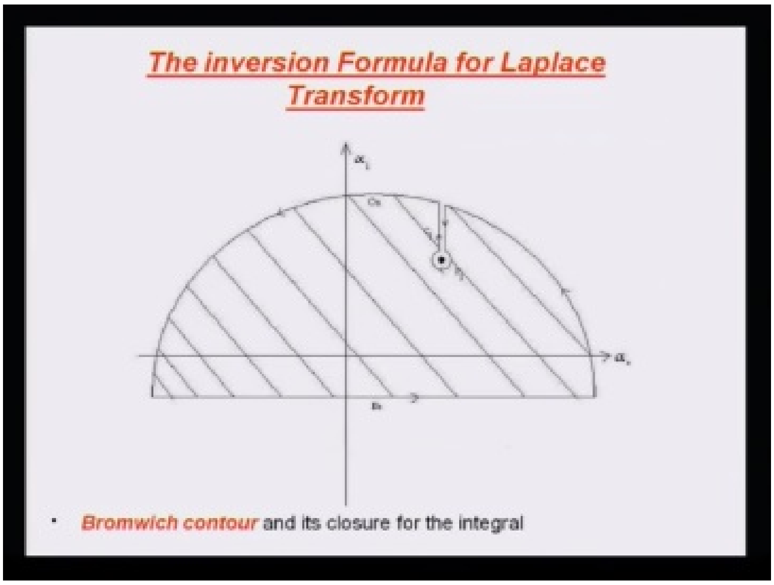

Then, what happens is we do this.

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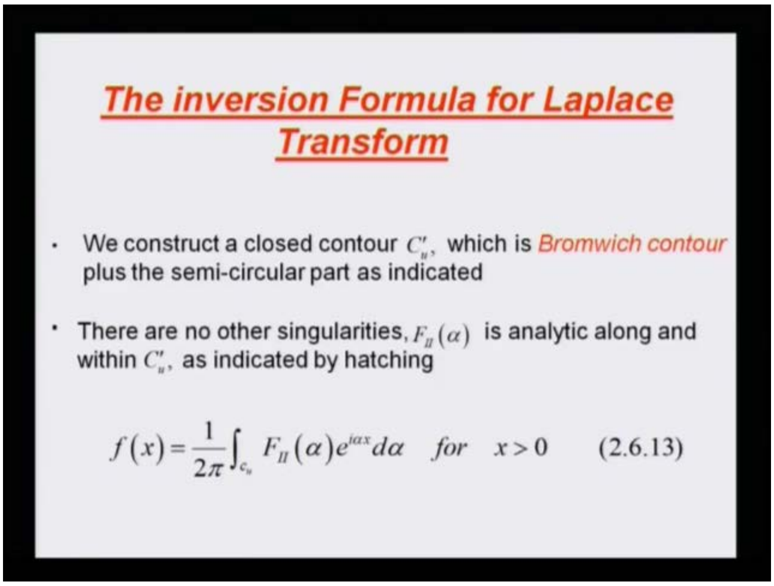

That is what we have done in this.

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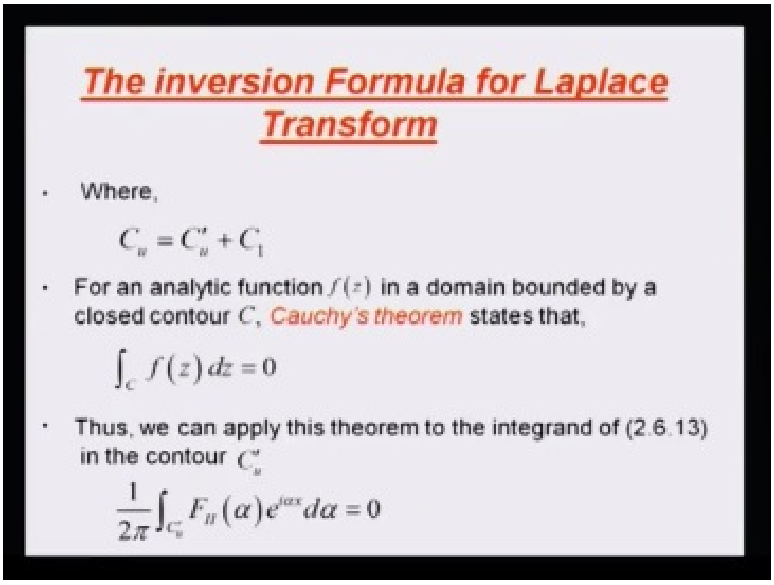

And, what we find, that this is what you get; that C upper as I told you, consist of C 1 and the rest of it. So, C u prime also includes the Bromwich contour. Now, in this C upper, f z dz is 0. So, this is what we are talking about. Here of course, z is replaced by alpha. So, this is what we get. And, please understand, f z is nothing but f II alpha times e to the power i alpha x; this whole thing. The whole thing is the f z there.

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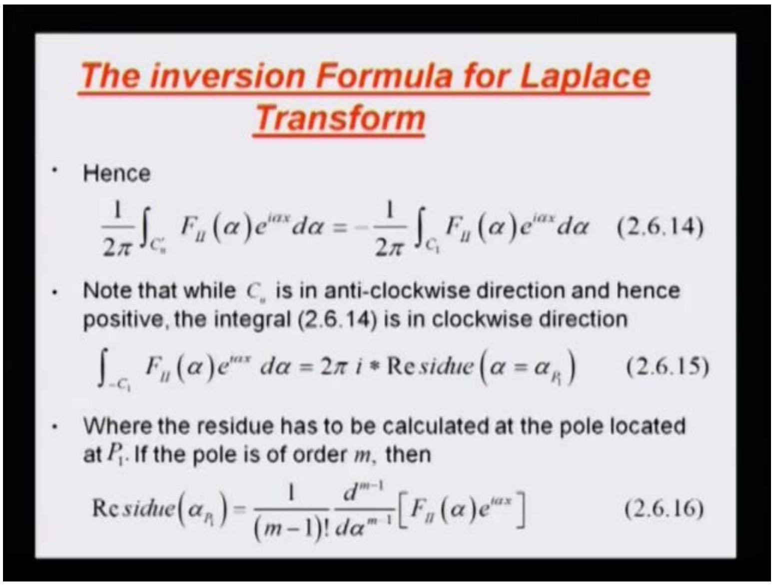

Now, this part, I did not do cleanly. So, I thought I will do it today. So, what I did now, I have this integral evaluated over C u prime plus the integral evaluated over C 1 equal to 0. So, I put this one on this side. And now, when I do this, I notice that C 1 is in clockwise direction. So, what I do is, I can make it plus and make this counter clockwise. And, that is the definition of positive quantity. So, we have this.

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Basically then, the contribution coming from (Refer Slide Time: 07:19) B r plus this is nothing but equal to the contribution coming from this. And, you know that this quantity (Refer Slide Time: 07:31) – what I have done here, I have changed it to minus C 1. What is this? This will be 2 pi i times the residue of this calculated at alpha at the pole. And, this is the definition how you calculate the residue. So, I could have different order pole. If I have say mth order pole, what I do? I take the function; I differentiate it m minus 1 times and of course, divide by 1 by m minus 1 factorial; and then, substitute alpha equal to alpha P 1. That is the way to calculate the residue. So, once I do that, I am done. And, this case that we have just now talked about, we have a simple single pole.

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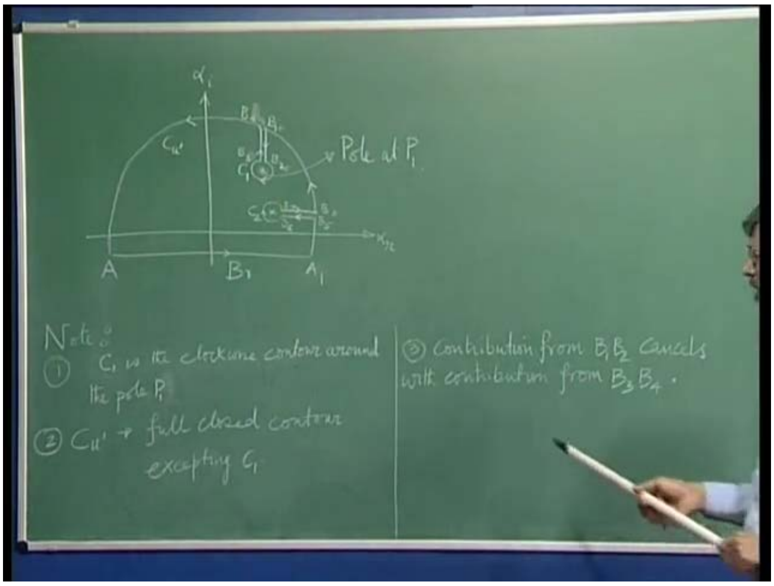

But, you can now see, you can extend the logic if you have let us say, more number of poles; suppose I have another pole here, what I could do is, I could just simply do what I have done there. I could just simply go there and circumvent this way and then I will go this way. So, I could call this as say B 5 and B 6; and, this I could call it as C 2 and let say B 7 and B 8. And once again, you can see that B 5 B 6 and B 7 B 8 will cancel out and this will be now this. So, then, what will happen? Then, rest of the contour would be given by the residue calculated at P 1 and P 2. So, you can just simply add it up.

Now, if the contribution coming from the semicircular arc is zero, then of course, whatever we have calculated is nothing but the integration over the Bromwich contour. We will come to that. This is an intriguing development that is going to take place. We want to see what happens. So, at this point in time, what we are saying, that if I do this, I will get this. Now, what happens is, any pole lying above corresponds to the downstream propagating. That is one thing we are talking about.

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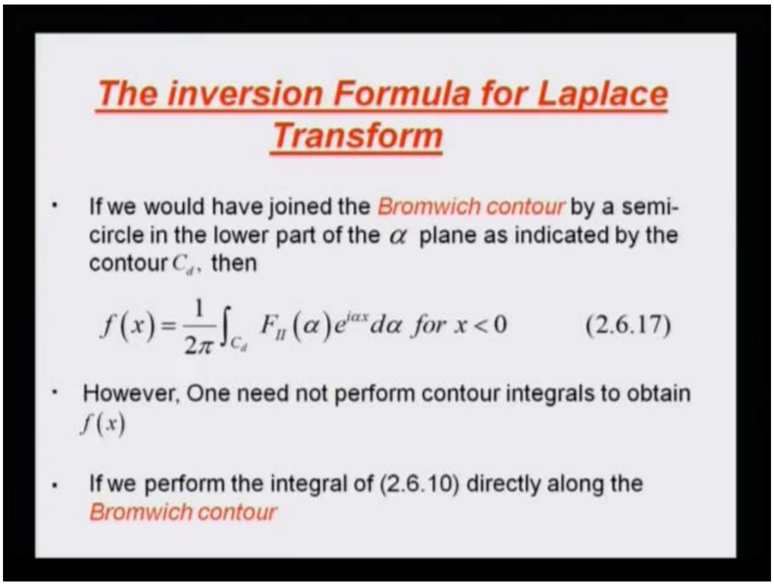

And, instead of closing it on the upper side, I could have also closed it on the lower side. That is what we are calling as C d. And then, whatever I have done here, I could have done it there also. But, there those poles would correspond to contribution coming in the upstream propagation. So, this is something that we need to remember, but you do not really need to worry about either closing the contour on top or closing the contour on the bottom, because this (Refer Slide Time: 10:45) circular arc – if Jordan's lemma is valid, it does not contribute anything. However, as I told you, this semicircular arc with a radius going to infinity is nothing but half the point that infinity. So, what do we call? If the function is not 0 there, we call it the function to have an essential singularity. You have heard of that essential singularity. For example, e to the power z, cos z, sin z, z going to infinity; do they disappear? They do not disappear to 0 when z goes to infinity. So, if I have such functions, then of course, Jordan's lemma is not going to be valid. So, in developing this part of the theory, we are assuming that there are no essential singularities. That is what Jordan's lemma means.

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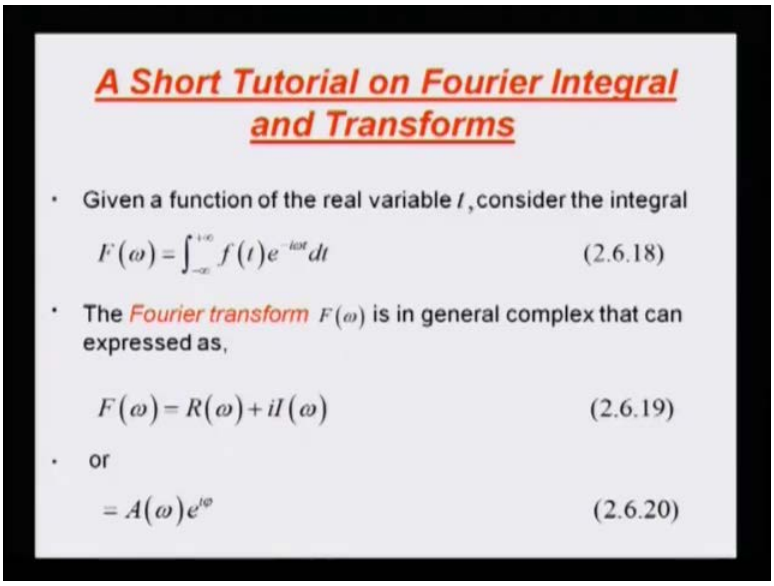

Then, this is what we started in the last class talking about familiarizing ourselves quickly about Fourier integral and transforms. And, that is what we said. If I have a function f of t, direct transform is F of omega, is given like this. And, this transform itself will have a real and imaginary part, which I could write it as R omega plus i times I of omega; or, I could write it in terms of the amplitude and a phase part. So, both are viable options. So, this gives us what we do.

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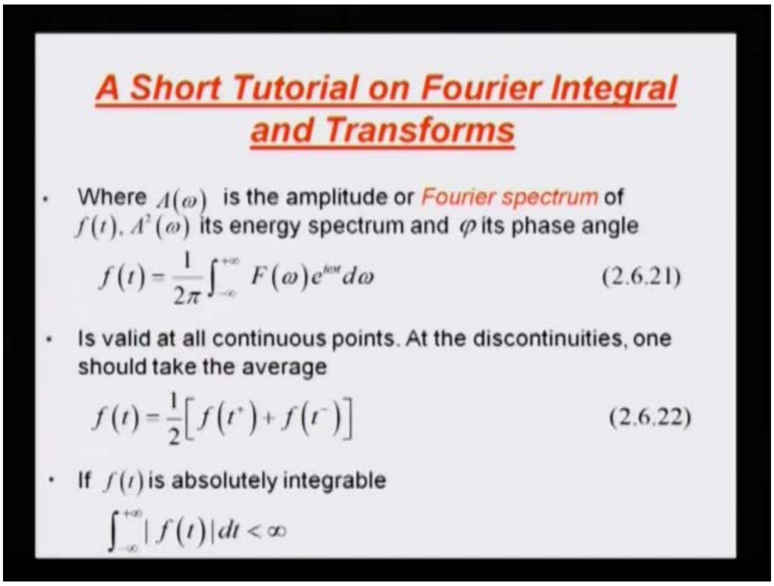

Now, we did spend a little time talking about this. This A of omega is amplitude or the spectrum; whereas, A square would be correspondingly the energy. That is what is called as the energy spectrum. And, phi is the phase angle. We talked about it that this representation is valid at all continuous points. At discontinuities, we take the right-hand limit and left-hand limits' average. That is what we did. And, we say that this function is absolutely integrable if I take the modulus of f of t and integrate it for all possible range of time. If it is bounded, then we know it is absolutely integrable. And, this is where we realized that not all functions are absolutely integrable and that is where we need to really make omega as complex. We realized; that is what here we did here also (Refer Slide Time: 13:29).

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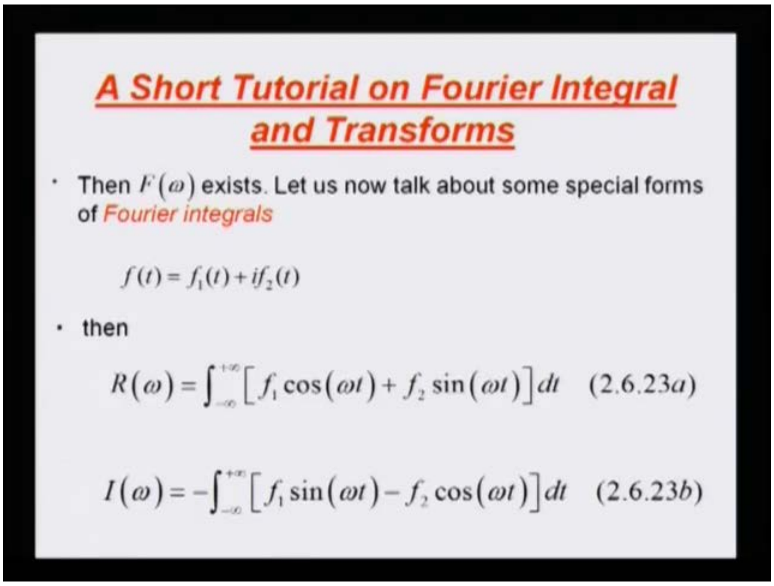

And then, what happens is, f of t also, I can write it like this. What does it mean? To have a real and imaginary part, think of f of t as some kind of a response of a system. And yesterday, we talked about time origin when I start the experiment. So, existence of f 1 and f 2 implies that I have a modulus (( )) phase shift. So, I can give an input of one kind of time dependence, but the output could lag behind, phase shifted. That phase shift relationship is given by the relationship between f 1 and f 2. Now, if we define f of t, the response in terms of a real and imaginary part, the real and imaginary part of the transform is given in terms of this. So, this is what we would be using often. So, let us familiarize ourselves.

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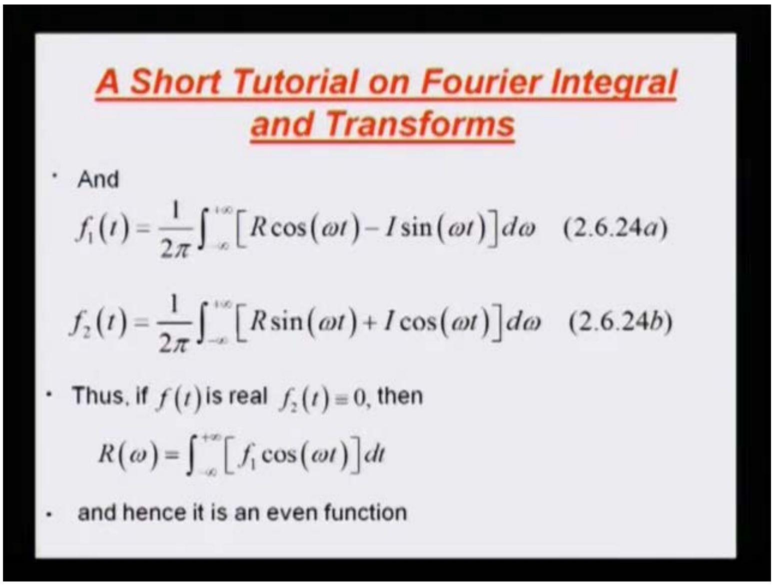

And, we can do similarly an inverse transform to get back this f 1 and f 2 in terms of R and I. This is what it is. For some reason, let us say, if f of t is real, then what we find from here that r of omega is given by this. And, it is going to be an even function if f 1 itself is even. Then, instead of doing the integral from minus infinity to plus infinity, we can do half range integration and multiply by 2 if f 1 also happens to be an even function.

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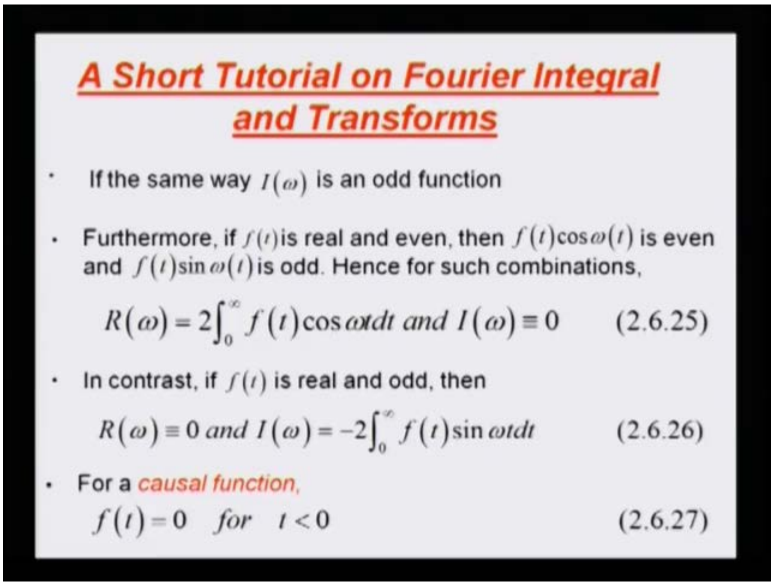

Same way, if f 1 is even, then I omega becomes an odd function. Suppose say f of t is real and even, then I have said that this part is even; this is an even function multiplied by even function, is even. And, while this one becomes odd, what happens? R of omega can be done like this; I of omega will be 0. In contrast, if f of t is real and odd, then I have the complementary picture, R of omega will be 0; I of omega will be given in terms of this. And, we talked about causality; a causal function is one that is equal to 0. And, from a receptivity point of view, this makes tremendous sense. If I have not started the experiment, the past cannot dictate what is going to happen in future.

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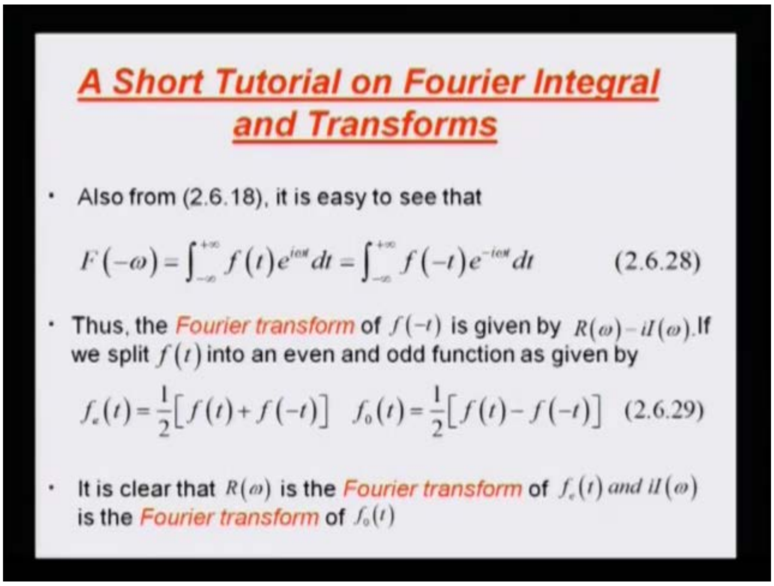

Now, we can also see, one interesting aspect is, if I replace omega by minus omega, then you can see earlier I had e to the power minus i omega t; so, it becomes e to the power plus i omega t. That would be simply nothing but f of minus t e to the power minus i omega t dt. So, Fourier transform of f of minus t would be given by the complex conjugate of f of omega. Or, if we can split the f of t in terms of an even component and odd component, how do you construct an even component? It is very easy; you take the average of plus t and minus t contribution; that will give you the even part; and if I subtract it, I get the odd part. So, this is one way of representing the same f of t in terms of even and odd component. You can clearly see that Fourier transform of this will be R of omega; whereas, if I take the Fourier transform of this, I will get i times I of omega. You can just simply substitute in the formula and we will get that.

In this today's class, we are mostly going to talk about some of these mathematical fundamentals. But, we will also try to connect it with what we intend doing.

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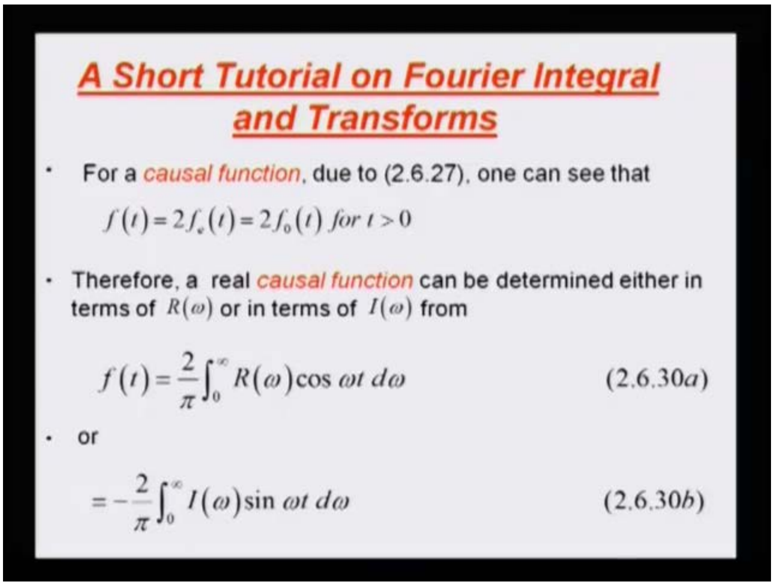

Now, we have seen, for a causal function, f of t will be nothing but twice of f even or twice of f odd. So, if I have a real causal function, then I can determinate it either in terms of R or in terms of I; either of the formula. When you do this, this is what is called as a cosine transform; this is what is called as a sine transform. So, Fourier transform is the most generic form. But, if you have a real causal function, you can get away with performing cosine or sine transform. And, you would note that in many of these utilities and packages, you do have an option of computing sine and cosine transforms.

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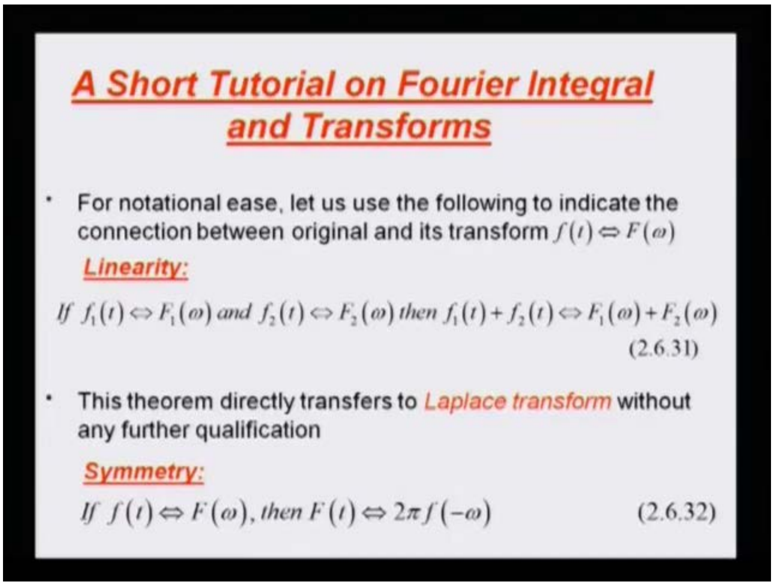

We did talk about this that if I have a function in the physical plane given by f of t and the Fourier transform in the spectral plane as f of omega, the correspondence between the two is given by this double-sided arrow. So, this is what it means; that they are related by the formula we have talked about. We talked about then two such functions, f 1 and f 2, which have their respective transform pairs. And, we showed the linearity property holds, because Fourier transform operation itself is a linear operation. It does not matter whether f of t is governed by nonlinearity or not. The transform is a linearity operation. And, this Fourier transform property directly transforms to Laplace transform. So, there is no such problem.

And, we have noted the symmetric property also that if f of t relates to capital F of omega, then I can actually construct a time dependent function, whose form is given by this transform. So, capital F of t is nothing but 2 pi f of minus omega. I am not doing it; you can just simply substitute in the formula and you will just get them right away. Or, you can look at that book by Papoulis. It is a fantastic book; one of the best book that one can cover across on this subject.

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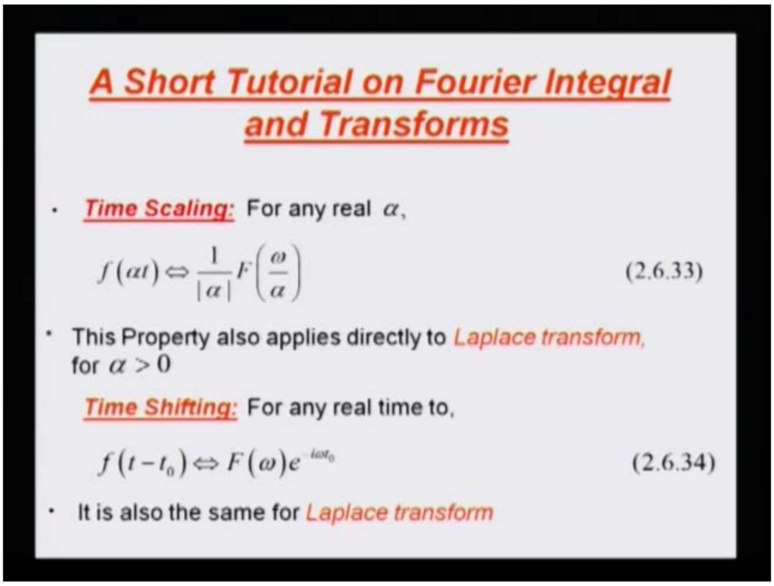

We did talk about time scaling. This also directly comes from the definition of the Fourier transform. And, what this is that if I stretch the time, let us say, alpha is more than 1, then I am taking each time and multiplying it. So, I am stretching the time. Then, the corresponding transform will appear like this. Once again, I will invite you to prove it yourself. And, you will find that this is a kind of a duality property. So, if I stretch it in the physical plane, it contracts in the transform plane. And, if alpha is less than 1, then I am contracting in the physical plane; it extends in the transform plane. So, this is the property that we readily also can see as an example if I have a simple periodic behavior of a function. So, that means what? In the physical plane, I have a signal, which goes from all possible time ranges. And, what happens in the transform plane? You just simply have a delta function. So, that is essentially fallout of this time scaling theory. So, you can benefit from it.

And, this property (Refer Slide Time: 21:45) also will directly apply to Laplace transform for alpha positive. Instantly, you are looking at it; that we are talking about alpha here is not that wave number; it is a real constant alpha. Now, there is additional properties; that also you can very clearly show that if your time origin is shifted from 0 to t naught, then you will see that corresponding transform is just simply multiplied by e to the power minus i omega t naught. It is that simple. And, we will use some of these properties in the context of Laplace transform also. It remains valid there.

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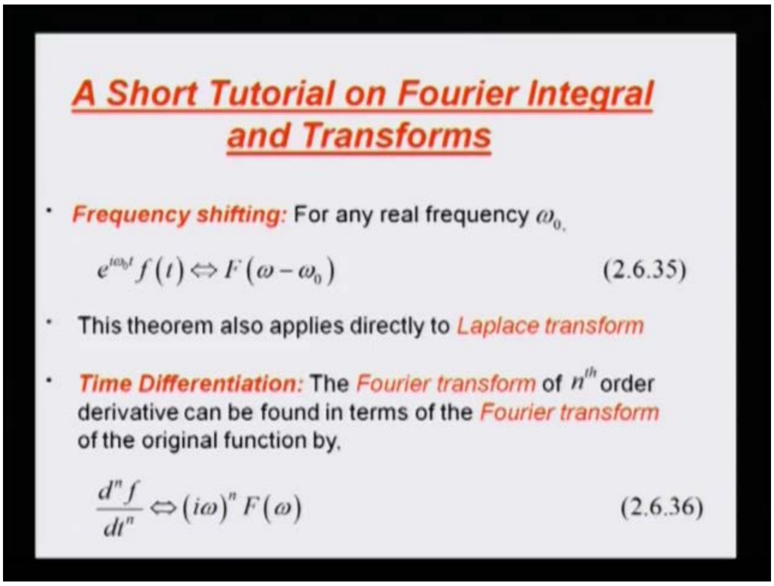

Time shifting is one such property; shifting of origin. The same way, if I shift my frequency also from omega equal to 0 to omega naught, the corresponding time dependent function is obtained by just simply multiplying f of t with e to the power i omega naught t. This also directly applies to Laplace transform. Now, this is something that we need to know the property of this. If I have a function f and if I differentiate it and then take its Fourier transform, then this is what we are going to get; say nth derivative of the function is related to F of omega by just simply multiplying by i omega to the power n. So, this is something that we can do.

Now, if you are imaginative enough, you can realize that if I replace t by x, then what will happen. I am talking about some kind of a spatial gradient. And, we will make use of this property very often as you would see that when we talk about distributions like source, sink, doublet, etcetera, there we will explore this possibility.

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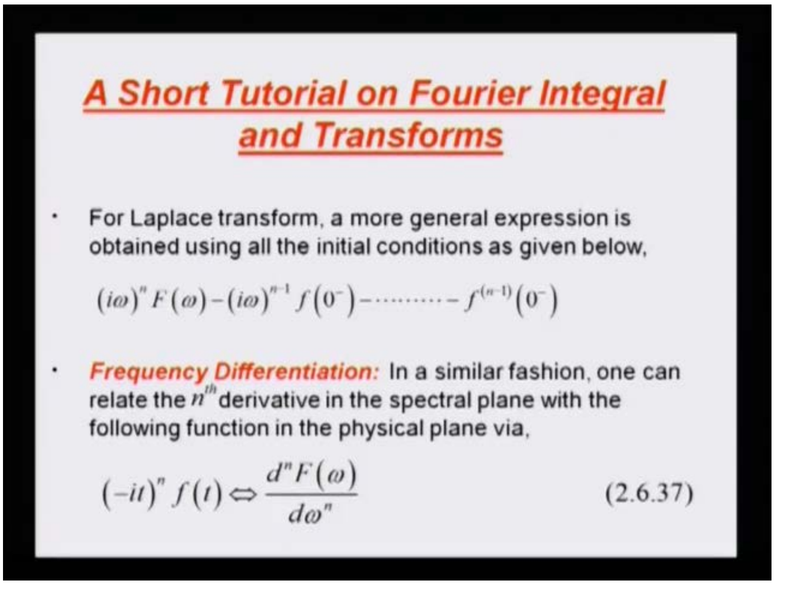

In timeframe, we can do this, but for Laplace transform, what happens? We need to worry about all kinds of initial conditions. They are all given here in terms of… This is what we had. Now, you have to add the first the function itself at t equal to 0; then, we will have the first derivative, the second derivative, all the way up to n minus oneth derivative evaluated at (( ))

There is this other nice property of frequency differentiation. We take F of omega; differentiate it n times with respect to omega. We can show, it is a corresponding original, is simply nothing but f of t times minus i t to the power n. These are all nice properties we can actually create a sort of a basket of Fourier transforms and its originals. And, that is what you can see in many standard books on Fourier transform.

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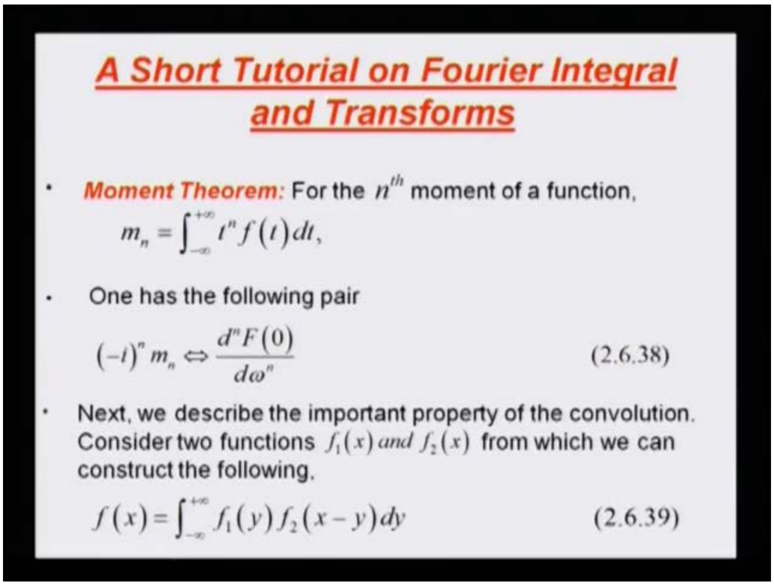

There is one more thing that we often talk about; this is quite often used, is the moment of functions. So, if I have f of t and let us say, I am talking about the nth moment, that means, I am multiplying the function at t to the power n, and then, I am performing the same integral. But, please do understand, there is no e to the power i (( )) Here, this is just a simple moment here. Then, what happens is, I have the Fourier transform f of t is F of omega.

Now, if I take that Fourier transform m with subscript n, this is what I get. So, this is quite often used actually for interpreting experimental data. Specially, in the context of turbulent flows, people try to figure out how various moments are, what is the significance of moments in the context of random signal – they will give you the various statistics. The average is the first moment; then, the RMS is the second moment and so on so forth. You have skewness, kurtosis, and so on and so forth. They all come from there. So, if I know the function or if I have some knowledge of the function and its Fourier transform, I can actually measure those moments in the lab and can see how the actual function is going to be.

In the context of this moment theorem, we can talk about a quantity called convolution. What is convolution? Say I have two functions: f 1 x and f 2 x; then, I can construct a function f of x, which is defined like this. So, I take f 1 of a dummy variable y and f 2, whose variable is now x minus y; I integrate over all possible ys from minus infinity and plus infinity. Then, the right-hand side is essentially going to be the function of x alone; and, that is that we are calling.

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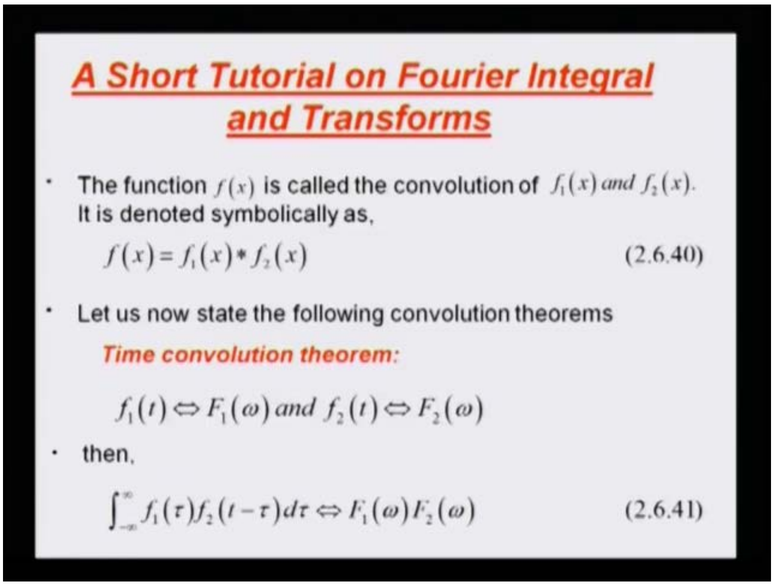

This f of x has a property with the transforms of f 1 and f 2 and we will see that; it is given here in this slide. But, first of all, we note that there is a symbolic notation for the convolution; f of x would be written by f 1 of x star f 2 of x. This is just notational convenience. Then, what happens is, we can talk about a convolution theorem. The convolution theorem states the following that if I have two functions f 1 of t and f 2 of t, corresponding transforms are F 1 of omega and F 2 of omega. Then, the convolution of f 1 and f 2 is simply nothing but the product of the transforms. So, that is why convolution plays such an important role; that the convolution in the physical space is equal to the product of the transform in this spectral space. This is what is called as the time convolution theorem.

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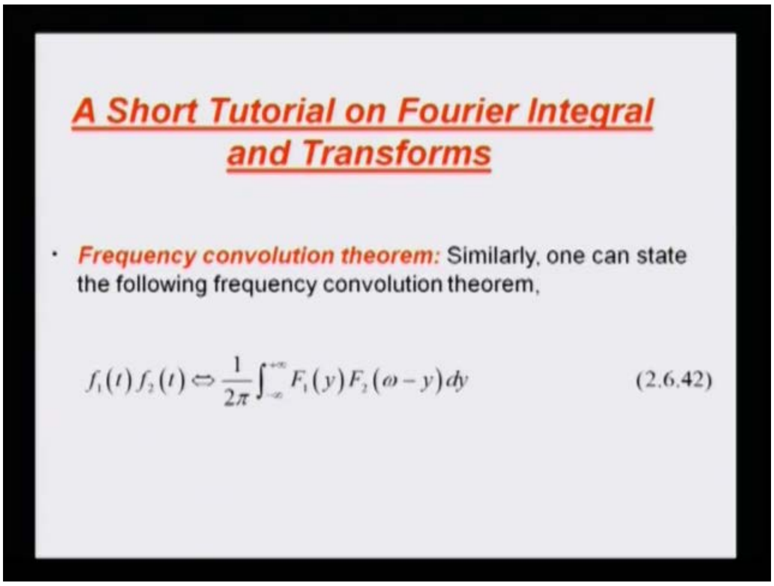

Similarly, you will also have a frequency convolution theorem. Frequency convolution theorem basically tells you that you define a convolution in the frequency plane; and, that has an original, which is just simply nothing but product of f 1 and f 2. So, these two are quite useful theorems, which we will use them as we go along.

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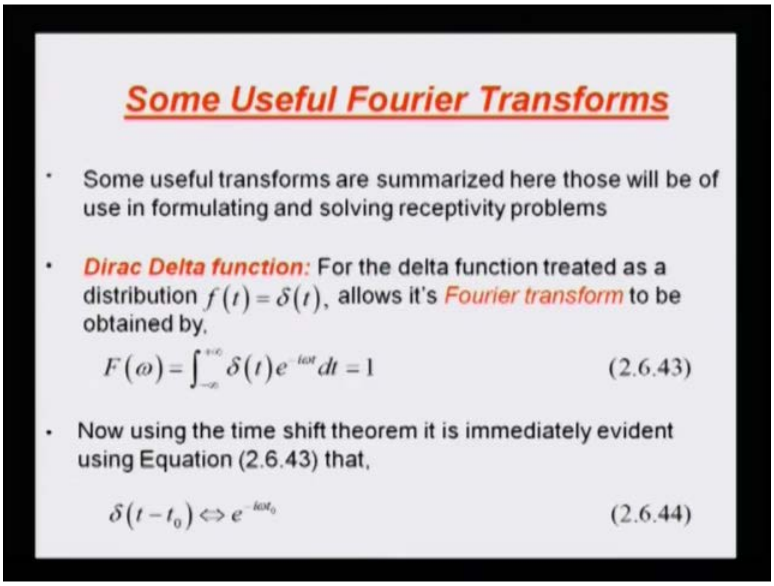

And, let us now talk about some things which are of going to be of use in our study. The first and foremost is of course, the Dirac Delta function. We do not want to go through the usual way of talking about the continuous function, etcetera; it is better that we adopt the distributions. And, the first element of distribution is of course, the delta function. You know its property; its property is that if you perform this integral, the integrand may take an unlimited value, but the integral will be limited. So, if I put it like this, what does it mean? Delta of t is 0 when t is non-zero; when t equal to 0, it is just equal to 1. So, if I put that here and I perform the integral, it becomes 1. So, this is what we have talked about in the past when we are discussing about frequency response versus impulse response.

This property as you can see, that if I create a delta function in the physical plane, I am just giving an excitation at t equal to 0; that fills up my frequency plane completely. And, they are all rated equally; there is no bias. So, if you are trying to look at let us say the natural frequency of the thing, you excite the whole system with all the frequencies and the system will pick up its natural frequency. So, that is why we said in the last class also that why impulse response would be preferable over frequency response.

Now, what you can do? This was for when the impulse was given at t equal to 0. But, suppose I give the impulse at t equal to t naught, then I can do the time shift property. Time shift property – what does it do? The corresponding transform – instead of 1, it will become e to the power minus i omega t naught. This is what just now we have written down. So, you can now see that we can talk about a dynamical system, where you could have let us say the delta functions not applied once, but in a sequence. And then, we can add all of them up and we know their transforms as given by this theory.

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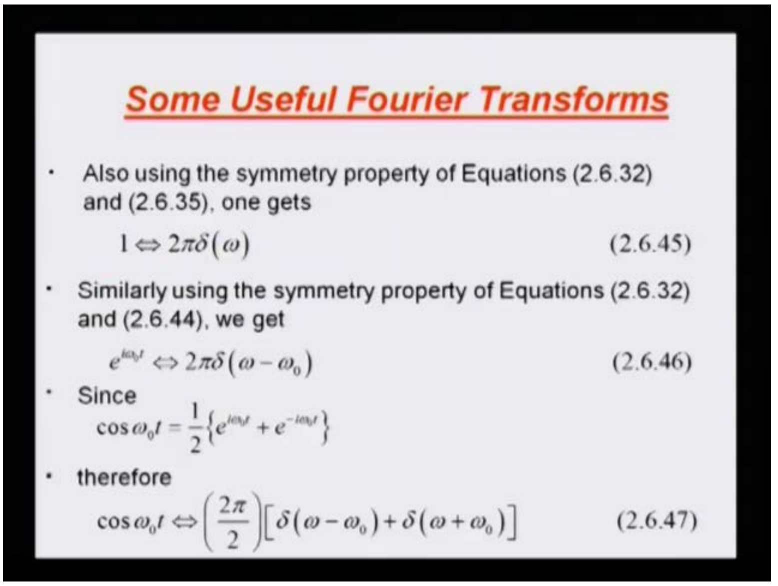

Now, we also can use the symmetry property. You recall that we had talked about – if I have capital F of omega, I can construct a capital F of t. So, we can do that, because we have seen, if I use a delta function, the transform is 1. Now, let us talk about – if I have a constant function in the physical plane, it is 1 everywhere; the corresponding Fourier transform is given by this. This is the symmetry property. So, this is the direct application of the symmetry property.

Now, what else we can do? There are lots of things we can do interesting enough with all those properties. Frequency shift theorem is applied here. If I shift the circular frequency from omega to omega naught, then I will get just simply e to the power i omega naught t. But, we also know what? Cosine omega naught t is not nothing but half of e to the power i omega t plus e to the power minus i omega t. So, then, what happens? I can see the correspondence; this function would have this (Refer Slide Time: 32:54). This comes from here; delta of omega minus omega naught will give me this multiplied by 2 pi. This 2 is of course, is there; and, this part will give me delta of omega plus omega naught. So, basically then, exciting a system by a real frequency, real function, cosine omega naught t is equivalent to in the frequency plane giving two delta functions symmetrically located about the origin at plus minus omega naught about the origin. That is what it means. So, that is the way of interpreting the application of a cosine function.

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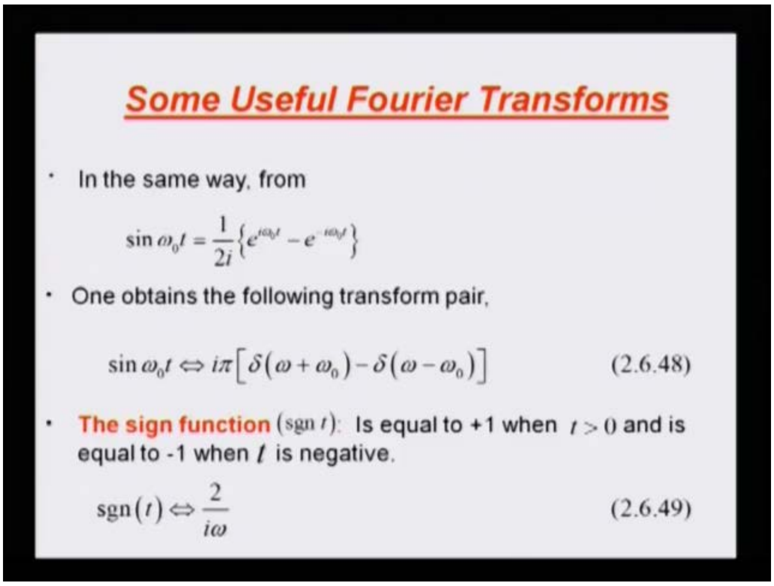

You can do the same thing with the sine function; only thing is note that there is an iota here in the basement. That will give you this relationship between the original and the transform. This also is nothing but two delta functions, but instead of average there, we have a minus here. So, that is what it comes.

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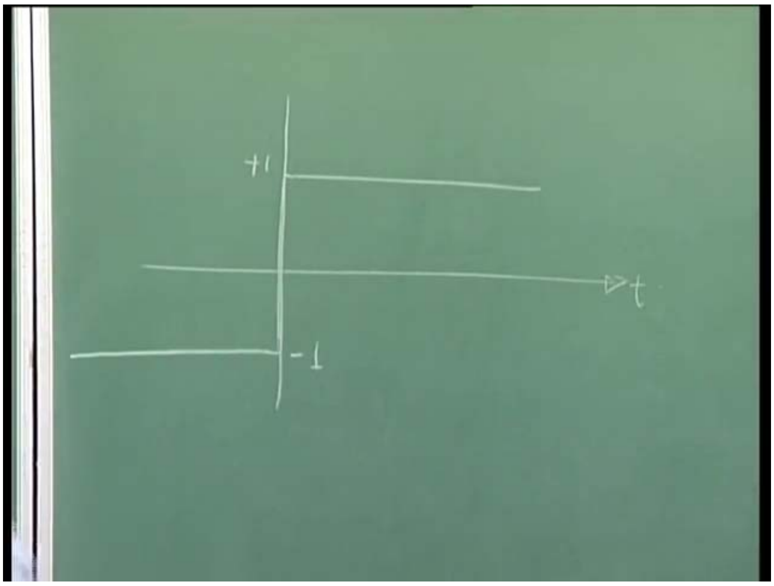

The next thing that we would like to do is what is called as the sign function. Sign function is interesting; it is like this. Suppose I have the time axis like this; the sign function is like this. That for negative t, it is minus 1 and for positive t, it is plus 1. So, this is minus 1; this is plus 1. So, that is what is called as a sign function or signum function. And, it is indicated by this (Refer Slide Time: 34:40) sgn (signum) of t; and, its transform is 2 by i omega. It does not fallout readily. So, let us see how we can do it.

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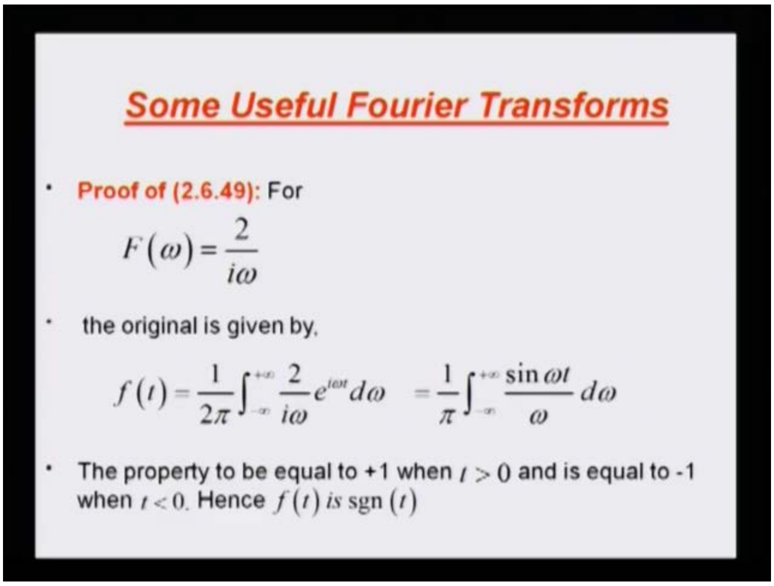

Basically, let us do the other way round. Let us say I have this F of omega given by 2 by i omega. So, I can construct the original from the property. So, I just substitute instead of F of omega, 2 by i omega. So, I get 1 over pi like this. And, this is a very interesting function. This is like the sine alpha x by x dx integrated over all range. And, what happens is that alpha – if alpha is positive, then this integral becomes plus 1; if alpha is negative, the integral becomes minus 1. So, that is what we are saying. So, in this context, what will happen? Here integral is over omega. So, if t is positive, this will be plus 1; and if t is negative, this will be minus 1. And, that is what we have shown here. So, basically, we have shown that if F of omega is this, then the corresponding time domain function is like this (Refer Slide Time: 35:58).

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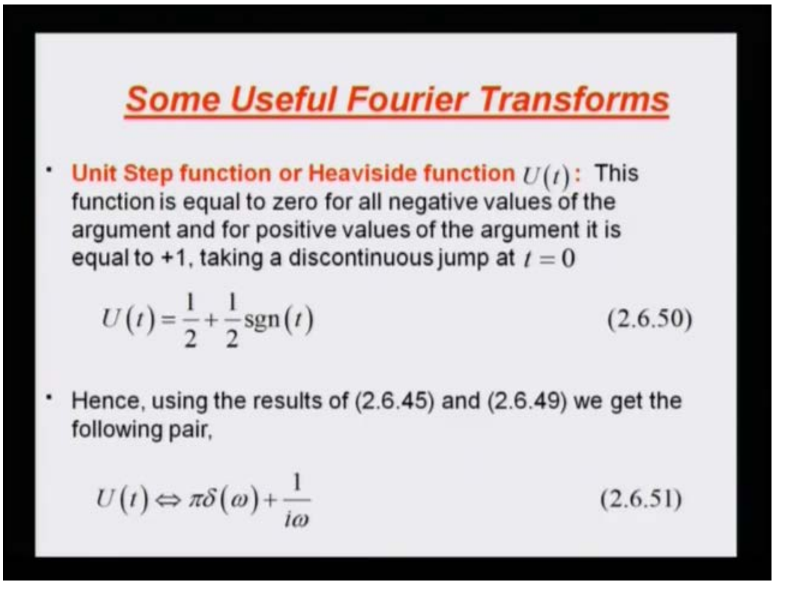

So, that is what we do to consider as useful, because we can use it to define the Heaviside function.

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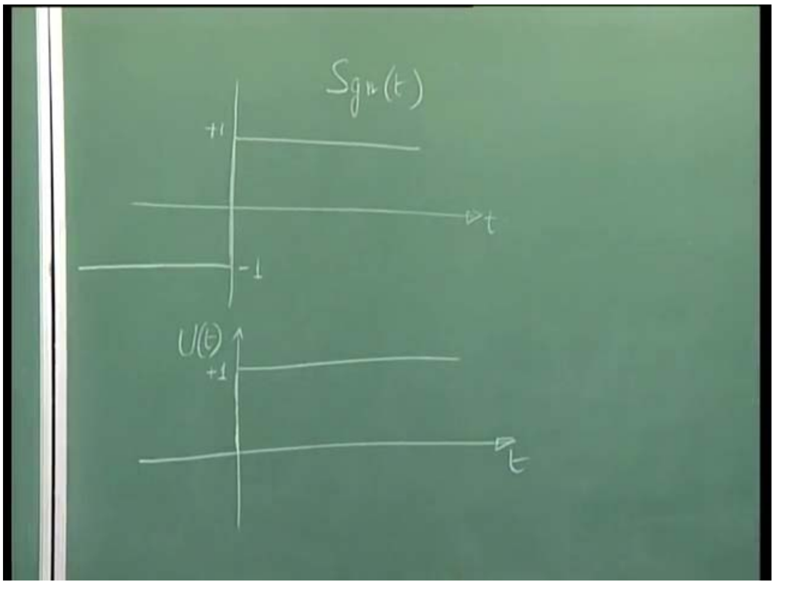

So, this is the signum function. And, what is the Heaviside function? That also you know; that Heaviside function is this; that it is 0 for all the negative arguments; then, it becomes 1. And, this function – we are calling as U of t. Now, you can see that these two are somewhat related. What I could do is, I could shift it up; if I shift it up, then this becomes plus 2; then if I divide by 2, then I will get this.

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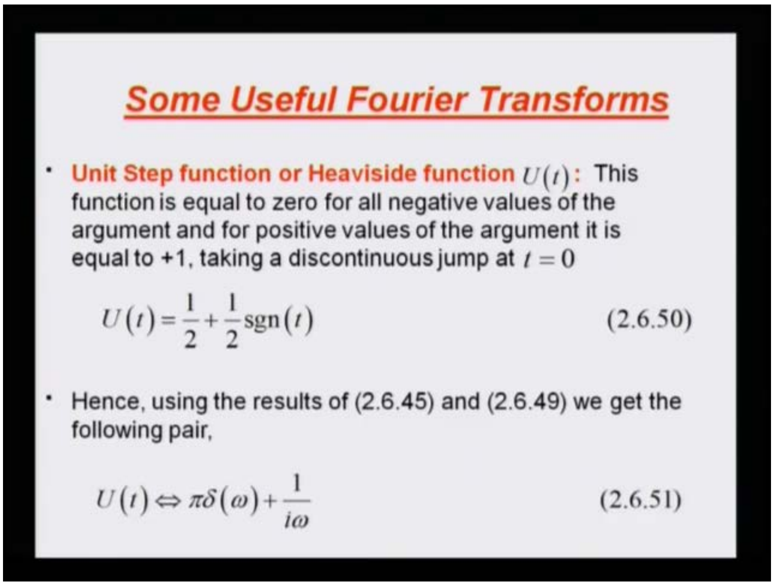

So, that is what we have done here. I have shifted first by 1; then, added the signum function; the whole thing has been divided by 2. So, that Heaviside function is like this. Once I have this, I can get its Fourier transform or Laplace transform. That is easy, because this is the constant; you recall that we just now talked about the symmetry property. So, if I have a constant quantity, that it is nothing but a delta function in the transform plane; and, signum function – we have just now seen; it is 2 by i omega, but due to this factor (Refer Slide time: 37:45) half, we have this. So, this is where we are.

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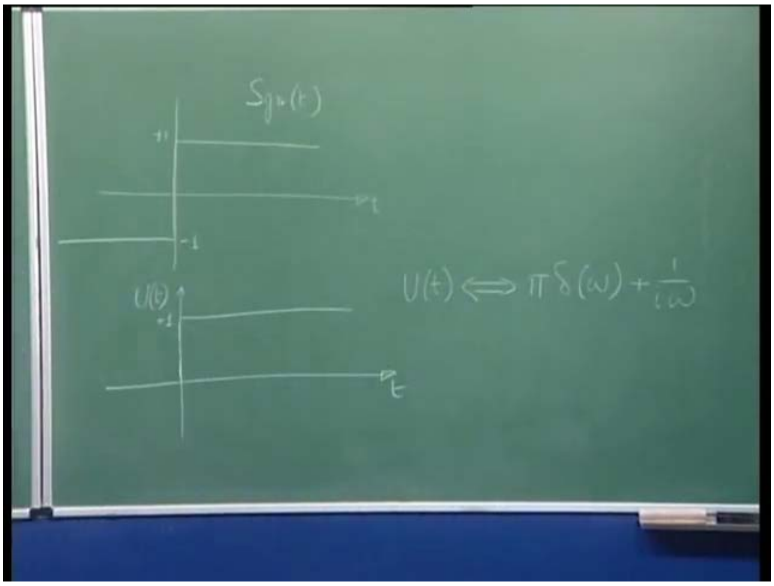

Now that we know what this is, what did we get? U of t arc has this transform pi of delta omega plus 1 over i omega; that is what we saw. Now, what we could do is, we can use a frequency shift theorem.

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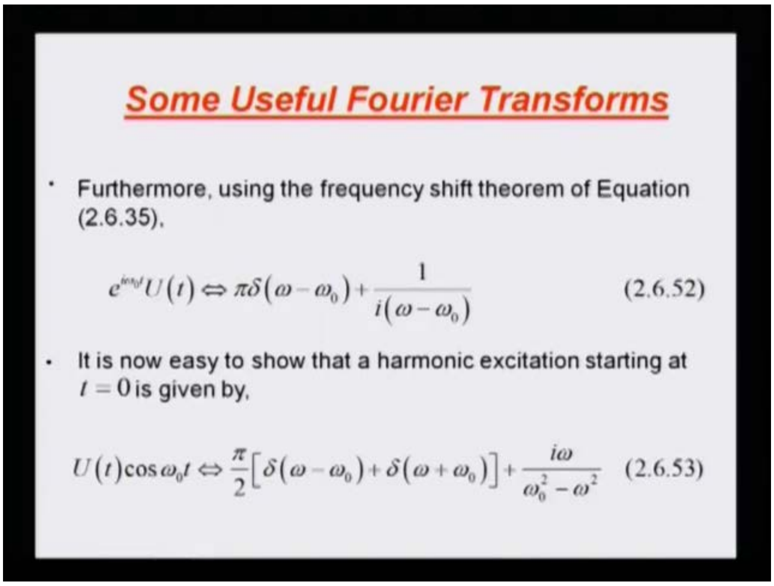

Then, what will happen? If I take U of t and multiply it by e to the power i omega naught t, then I will shift the origin from omega equal to 0 to omega naught. So, then, this argument will become this and this will be this. So, you can very clearly see. What is the utility of this? This is very important. Now, go back and think about your Schubauer-Skramstad experiment. What did they do? They took a ribbon vibrated harmonically. So, that frequency omega naught was given. And, they did start the experiment at some finite time; it is not like something which had gone on from minus infinity to and it is going on forever. Although for mathematical expedience, sometimes people do that, but we realize that in a real experiment, we need to invoke the Heaviside function, because we need to create a sign post for time; that it has started at this time. And, that is our t equal to 0. Then, we are basically doing in this and this is what it is.

Why we are doing this? Because in the physical experiment, we have started at a finite time; and, when we apply let us say, the governing equations, that we will be doing in terms of Orr-Sommerfeld equation, which was written in alpha or omega plane. If we do that, then we need to derive the conditions – those boundary conditions that we are talking about – in the spectral plane. And, that is what we have established here. So, if we are squeamish about that and we say look, I would rather do it with a cosine function; then, the input is like this – U of t started at t equal to 0 and cosine omega naught t. Then, of course, what will happen here? I will have to add up; remember, the e to the power i omega t plus e to the power minus i omega naught t divided by 2. So, that will give me this part (Refer Slide Time: 40:18). And, the same thing we will do it there and then we get this simplification. So, this is what we can do.

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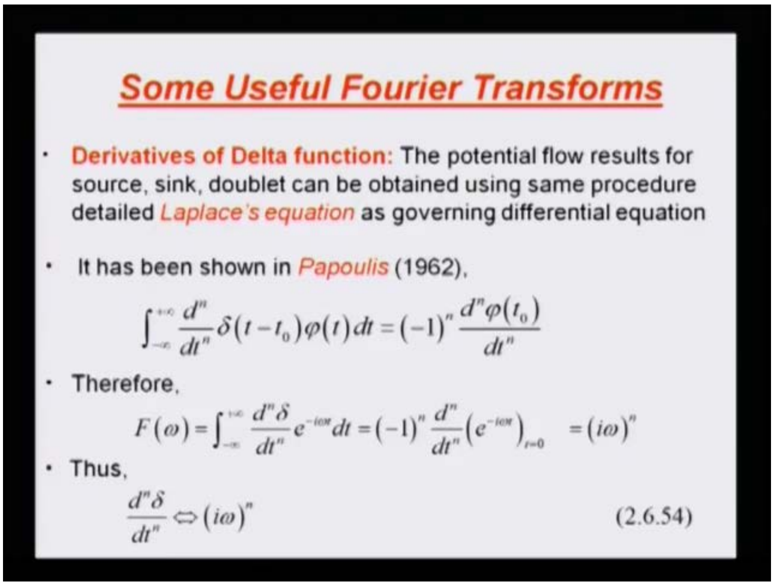

I think we have done quite a bit of things. I will just simply state one observation without doing much about it. Maybe some of you could do it later, is in your text book on fluid mechanics, you have been told about what is the stream function or the velocity potential due to a source or a sink. What are those sources and sinks? They are nothing but the delta functions. So, if I look at the actual problem, can I solve the problem? Because the governing equation is what? Laplace's equation for phi or psi.

Now, if I give a boundary condition that I am putting a source or sink at say the origin, then I can calculate the field. So, that is what I made this observation that the potential flow results for source, sink, doublet can be obtained using the same procedure by using Laplace's equation as the governing differential equation. We are also not going to talk about it, but you can take a look at Papoulis book that if I take this delta function, time shifted, and differentiate it n times, and multiply it by some phi of t, and integrate over all possible range of time, we get this (Refer Slide Time: 42:08). So, what I could do is, I could obtain the Fourier transform of nth derivative of delta function. And, that if I call it as F of omega, then use this relation here. Now, you can see, here phi of t is this e to the power i omega t. So, that will be minus 1 d n dt n of phi of t at… So, t is 0. So, what do I get? I get this.

What is the implication of this? (Refer Slide Time: 42:51) Implication of this is, if I have a source or sink, I use delta function; but, if I have a doublet, I have the first derivative of the delta function. So, it is basically having a source and sink brought close together. So, that is like the derivative of a delta function. So, if I look at the first one, then I will get the result for doublet. And, people working on acoustics problems do use all kinds of combinations of these singularities; they use quadrupoles, octopoles, etcetera. So, they are nothing but those higher derivatives of delta function. So, you can actually use some of these.

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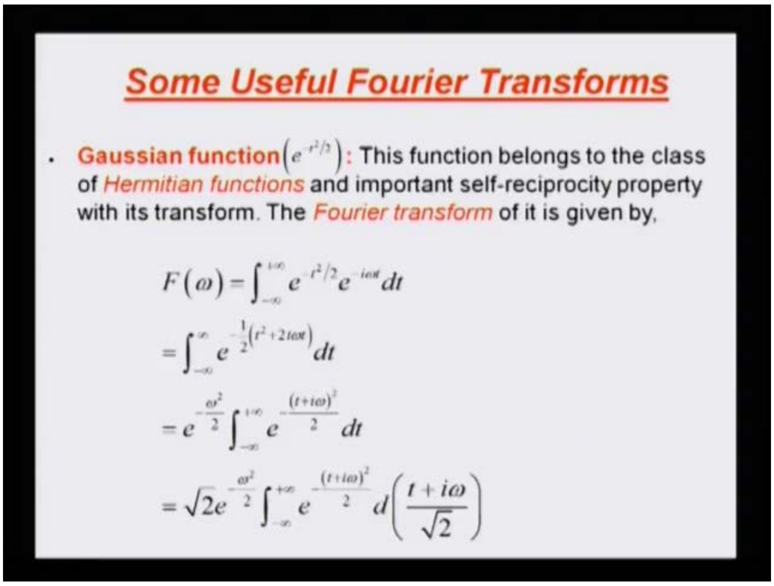

There is one very interesting function, which we often like to use. And, that is the Gaussian function. Gaussian function or sometimes, they are also called the Hermitian function. What is the property of this? That is displayed here. Property of the Gaussian function is I can try to obtain its Fourier transform. So, what I would do? I will take the function; multiply it by e to the power minus i omega t and integrate. Then, what happens here? If I add these two exponents together, then I will have minus half t square plus 2 i omega t. There is a minus sign sitting outside; minus half. So, what I could do is, I could write it as kind of exact square. So, that will be t plus i omega whole square. So, I have basically then added this part up (Refer Slide Time: 44:36). So, I take it out. That is a constant. So, I can take it out of the integral, because it is a t integral. So, I get this. Now, what I could do is, I could do a little bit of manipulation; write this as t plus i omega by root 2 as the independent variable. Then, since I have put in a root 2 here, I multiply it there. So, then, what happens? This is what we are quite familiar.

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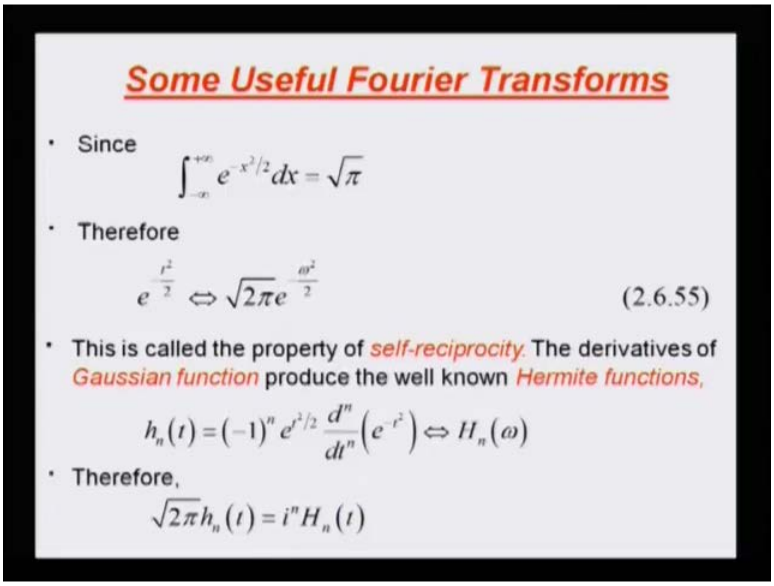

Once again by complex analysis, we can show that if I call this as i, I can multiply it by another i. And, there instead of calling it x square by 2, I can call it as y square by 2 and then i square will be nothing but e to the power r square by 2; and, dx dy – you can map the area into r dr d theta. You know 2 pi r into d theta and you get that. And, you can show that this is the result. So, what happens? That if I take the original as e to the power minus t square by 2, the Gaussian function, look at its Fourier transform; that is also e to the power minus omega square by 2. So, this is the property of self-reciprocity. The function reproduces itself upon taking the transform. So, whenever such a thing happens, they are called the Hermitian functions or Hermite functions. They are this generic equation. You can take the derivative of e to the power minus t square; and then, you can get a whole side of Hermite functions of different order n. And, the corresponding Fourier transform – we will call it H of n and one can show that root 2 pi h n of t should be i to the power n H n of omega. So, I think we have done with this part.

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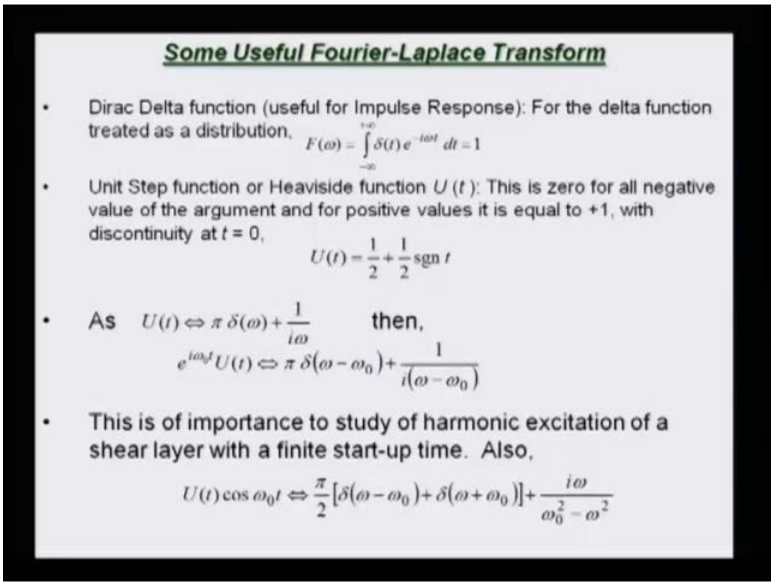

Now, what we could do is, we could go back to what we have been looking at. We are talking about the receptivity. We have seen through all of these. So, let us go over and there we are. See, we are now talking about Schubauer-Skramstad kind of experiments. So, there we give this kind of input. Now, we want to solve it as an excitation problem; not as an eigenvalue problem.

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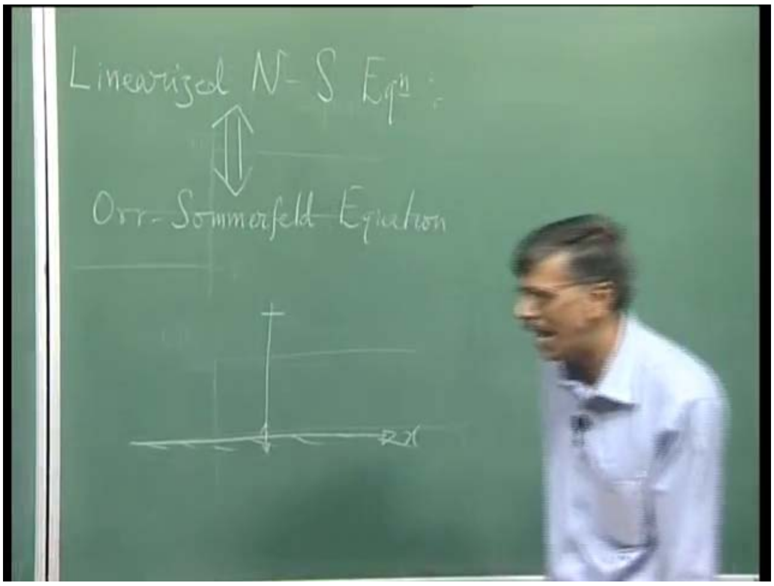

What is happening here is, now, we are going to solve the problem the way we have seen. We will take about small disturbance. So, we will be talking about linearized

Navier-Stokes equation. Linearized Navier-Stokes equation is rather easy; we have seen it. If we do a Fourier transform of that equation, what do we get? We have seen that; that is nothing but Orr-Sommerfeld equation. And, what happened? Orr-Sommerfeld equation involves let us say, I have this flat plate; I want to study this. Now, what I need to study? I need to study let us say, I take (( )) x location; what I would be doing? I would be solving Orr-Sommerfeld equation from some limit of y equal to 0 to let us say y going to infinity; like that. And, what is it that Schubauer and Skramstad experiment did? They vibrated a ribbon very close to the wall. So, we will just simplify it furthermore. We will say we will vibrate the ribbon at the wall itself. That is the corresponding receptivity experiment. So, I have a vibrating ribbon right at the wall itself.

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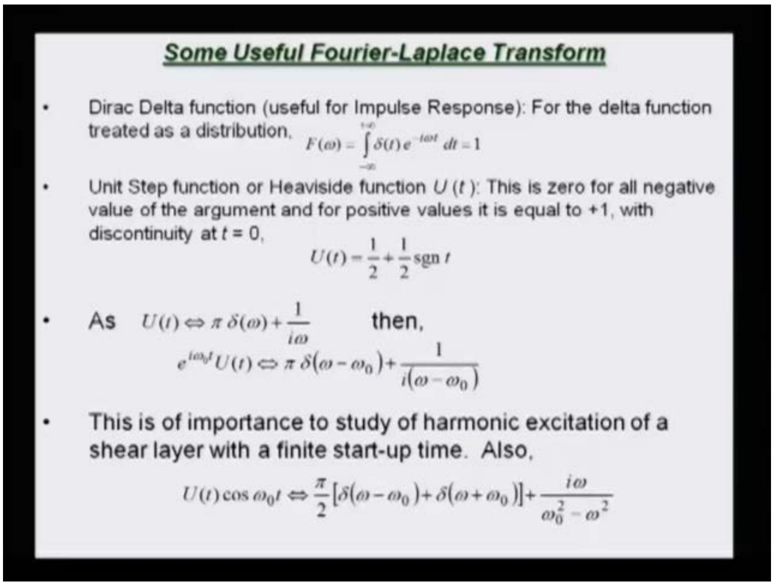

And, that quantity is given like this (Refer Slide Time: 49:30). Now what happens? I get this. Now, you can also see why we did talk about signal problem in the last class. In the last class, we said that if I vibrate a ribbon with omega equal to omega naught, response is also at omega naught. But, now what you are seeing here? The input itself has a contribution at omega naught, but it has also a contribution elsewhere. See, this could be a delta function; no problem. This is corresponding to what you may think of in terms of the signal problem assumption. Because you are doing it, exciting the system at omega naught, you are seeing the response through this part. But, what about this part? This part is not gone. So, this part actually decays around that, but still it is there. So, even though

I am exciting the system at a fixed frequency, omega naught, I am also exciting its neighborhood altogether.

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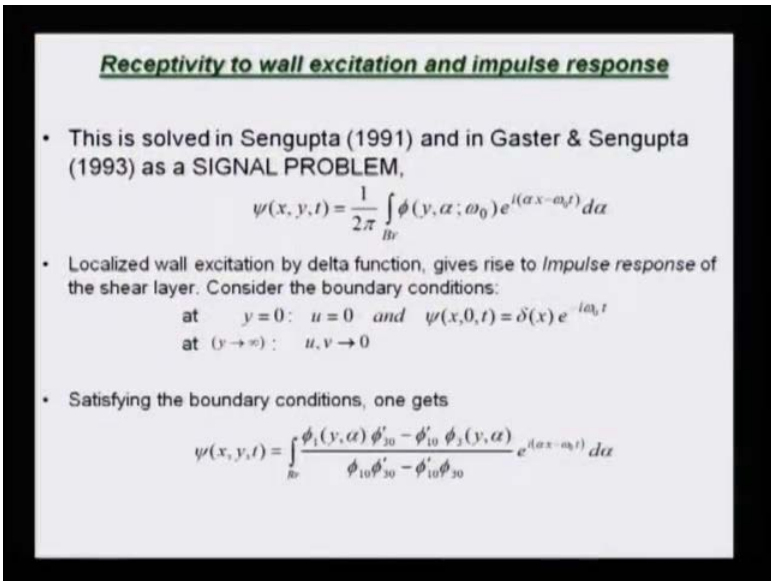

So, what happens is, signal problem assumption is not all that a great thing and this is what we realized when we tried to work on it. So, what we would be then talking about – instead of this signal problem, we may actually like to do here keep it omega and then we perform a second integral in the omega plane also. However, for the sake of understanding, what happens in the receptivity problem – let us assume that we are talking about a signal problem. And then, what we are doing, this is the type of our boundary condition. So, what I have done, at the wall, I am saying that there is no slip; u equal to 0. But, let say I am blowing and sucking mass at a frequency omega naught; and, I am doing it at a fixed location. Let us fix the origin there itself. That is why I am calling it a delta x; delta x minus x naught; there is no problem. We have seen the time shift theorem can be used.

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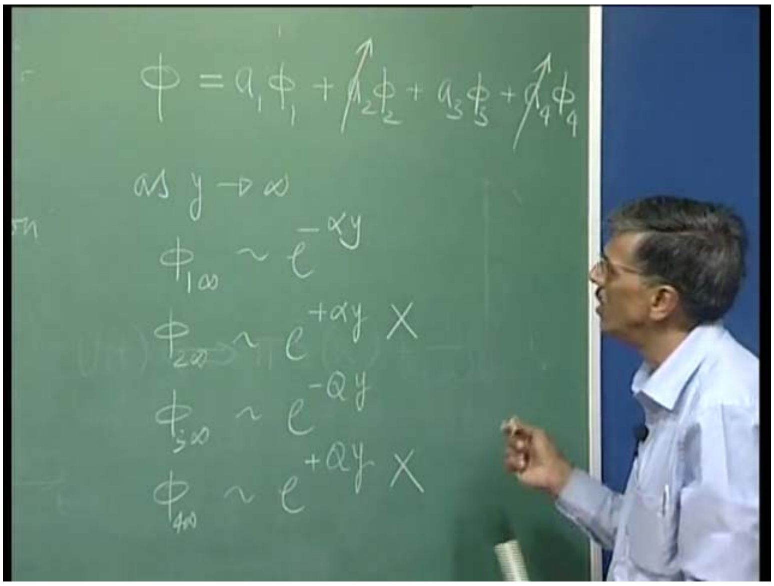

Now, what we have to do is, we will have to satisfy the boundary condition. How do you satisfy the boundary condition? If you recall that we have four modes for Orr-Sommerfeld equation; so, it is a fourth order ODE. So, I could talk about a 1 phi 1 plus a 2 phi 2 plus a 3 phi 3 plus a 4 phi 4. What do you know about these individual modes phi 1 to phi 4? We only know when we go outside the shear line; if we recall, that is how in defining the initial condition for compound matrix method, we evaluated those modes. You can substitute the condition of y going to y infinity. What will happen? U of y will become 1; U double prime will become 0; and then, we will have a constant coefficient ODE fourth order and we get those four modes. And, we saw that this one – as y goes to infinity, we noted phi 1, which I will now give additional subscript infinity, goes as e to the power minus alpha y; and, phi 2 infinity goes as e to the power plus alpha y; phi 3 infinity goes as e to the power minus Q y. You remember the definition of that Q square; so, you can take a look at that and this will be this (Refer Slide Time: 53:37).

Now, if you are talking about a wall excitation and if you say that real part of alpha is positive, then this is not admissible, this is not admissible, (Refer Slide Time: 53:49) because these are showing it to grow with y. So, we have to keep only in terms of this. So, how can this happen? All these kind of things can happen only if we switch off a 2 and a 4 term. So, we have two terms: a 1 phi 1 plus a 3 phi 3. So, what I will do is, I will just stop here and I will ask you to look at these conditions (Refer Slide Time: 54:17). Now, what happens is, we will satisfy the conditions at the wall. At the free stream, we have already done it; by excluding phi 2 and phi 4 contributions, we have already satisfied these two conditions. But, now, we can fix a 1 and a 3 by satisfying these two conditions and you will be able to get this equation (Refer Slide Time: 54:39). That is what I want you to take a look, but we will redo it later.