Instability and Transition of Fluid Flows
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Lecture No. # 13

In the last class, we made some remarks about the short coming of eigenvalue analyses and we proposed that instead, we would look at receptivity analysis of shear layer. We are still confining our attention to viscous instability mechanism, because that is all encompassing. The study would also include the inviscid modes as you have seen from the Orr-Sommerfeld equation.

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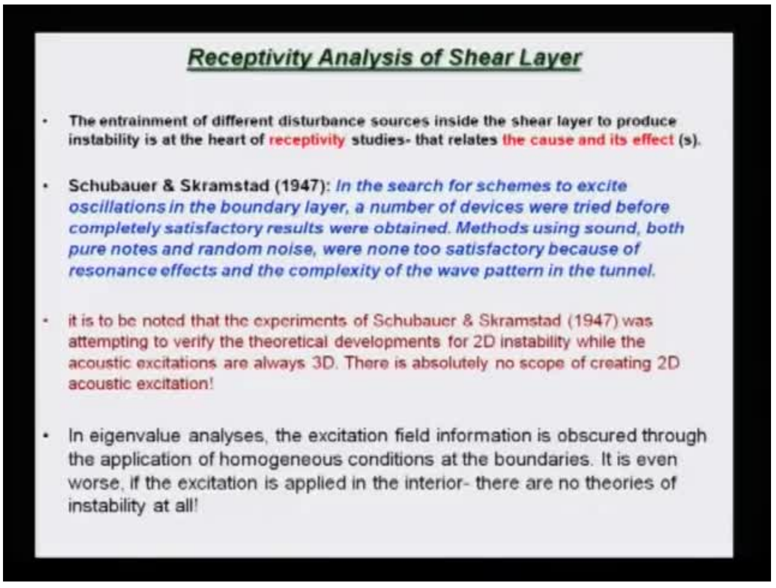

What you do in receptivity study is, basically relate cause and effect; that is what we say. That gives you a sort of leverage to work out circumvents of the problems, those were identified by early experimentalists like Schuhbauer and Skramstad. They wanted to find out how to work out the best means by which instability waves could be excited. And, this was what they had to say, that in trying to work out a method, they found that method using sound, whether it is pure notes or random noise, were not at all satisfactory. And, all they could do is, they could vibrate a ribbon inside the shear layer very close to the wall; not exactly at the wall, but very close to the wall. And, they could get those tollmien-schlichting waves.

We particularly pointed out that if you are looking for 2 dimensional instability modes, acoustics is going to be a very poor candidate, because acoustic excitation is inherently 3 dimensional. Nonetheless, eigenvalue analyses obscures your vision to give you some idea about causes itself. You just simply try to say possible effects through indirectly that, if there were disturbances, which would have been adequate to excite those, then what would be the effect? That is the kind of thing.

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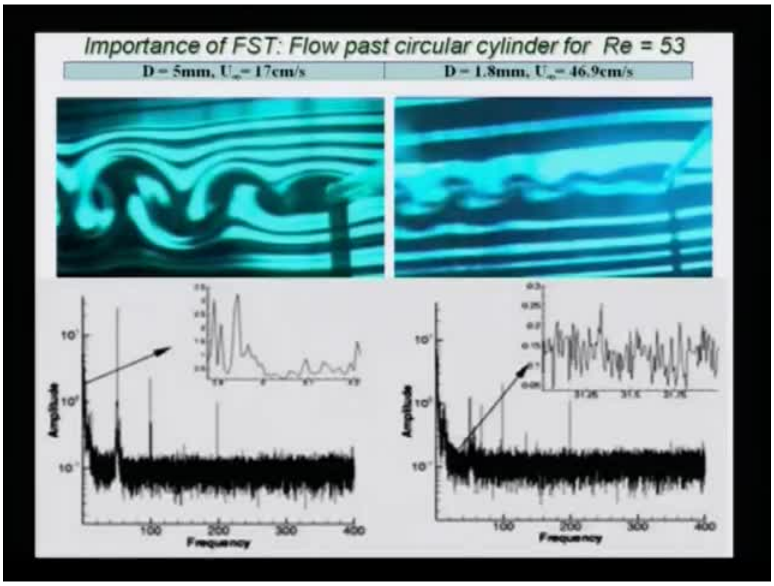

We did talk about free stream turbulence; how important it is, because the vibrating ribbon experiment of Schuhbauer and Skramstad showed that you could excite the TS wave by vibrating a ribbon inside the shear layer. But, suppose the disturbance is outside the shear layer; that is what the free stream turbulence is most of the time. Whether you are flying any aircraft or you are doing an experiment in the tunnel, it is what that you are getting in terms of the background disturbances.

Now, this result that you see in front of you is somewhat very intriguing and special, which we did few years ago. What we try to do, is study the flow past circular cylinder for a Reynolds number of 53. Why 53? Because earlier theoretical studies have revealed that if you keep your Reynolds number about 47 or 48 or so, then you see this vortex that is sheared at the back of a cylinder, circular cylinder. Now, this event is traced to the instability of the flow. Now, the question is – then, for a Reynolds number of 53, we should see a shedding and that is what one would like to do. At the same time last class, we did critic about characterizing experimental facilities in terms of a single quantity called turbulent intensity like Tu; what you define as a kind of a root means square disturbance; average disturbance in the physical plane itself. That kind of characterization falls a little short and that is what we wanted to highlight through this experiment. So, what we did? We fixed the Reynolds number and we took two cylinders: one of diameter 5 millimeters; another is diameter 1.8 mm. And, to keep that Reynolds number, in this case, the tunnel was set at a speed of 17 centimeter per second. And, in this case, (Refer Slide Time: 04:59) it was much higher; we required 46.9 centimeters per second to simulate the same Reynolds number.

Now, forget about all the things that you have been told in the first course in fluid mechanics about similarity parameter. You have been given a study diet of the idea that if you keep the similarity parameter same, flow would be same. Then, we wanted to prove that and that is what we did. And now, you can see, here for this case at higher velocity, you do see some eddies in the back. This is a prolonged attached eddy here; this at that tail end of those two eddies, gives rise to something like an unrelating excitation something like what we studied in Kelvin-Helmholtz mode. That gives rise to breaking up of those attached eddies and gives rise to weak unrelation. Now, it is a matter of interpretation. Is this like what we know of the Karman vortex street in a flow past cylinder? That is what you see in this other case. For this lower free stream speed, (Refer Slide Time: 06:21) what we notice that very clearly the vortices are sheared right from the cylinder itself. It is quite unlike this case.

Now, why these two flows are too different? That you cannot get from the understanding of that so-called Tu, because we are working on the same tunnel. So, we have that so-called same Tu. What you profitably can understand is you try to find out what is the nature of the back ground disturbance in the tunnel? The same tunnel should have the same background disturbance; unfortunately, it is not so, because given the different speed, I get different disturbance environment. This is what it looks like (Refer Slide Time: 07:18) for 17 centimeters per second and this is what it is for 46.9 centimeters per second; the same tunnel. And, what is plotted here is the disturbance amplitude versus frequency. And, how it is done? We have removed the model; we just simply run the empty tunnel at two different speeds. And, can you see the disturbance environment being so different? Now, this is the kind of problem that one does not expect to see. What you notice though that from the earlier studies, you can find out this strouhal number; the shedding frequency – that is what you may put a probe little far down, measure the signal, and you will see a characteristic frequency, time variation. That is what is called as a strouhal number.

Now, if you look at the strouhal number, that is a fixed frequency for this particular Reynolds number. And, what we did was, we just simply zoomed this spectrum around the strouhal number. And, in this case, you see this is the strouhal number (Refer Slide Time: 08:23). There is a peak. And, this value is something like about 25 to 30; in that range. So, at strouhal number, we have the background disturbance; that is the most robust. In contrast, if I look, for this higher speed case, I get the value, which is about 10 times smaller. We get 10 times smaller at the strouhal number. So, what happens? This tells you a very clear story that if you do not have the cause, you do not have the adequate effect. So, in this case, cause is strong; 10 times more power than this. And, that is why you are seeing vortex shedding. So, this is the motivation for doing receptivity. What you need is the adequate strength of the disturbance field. Then only, you will see the desired effect; otherwise, you will not.

Unfortunately, if you would have done instability studies, it will not distinguish, because you are looking at the same Reynolds number, same tunnel; everything same. So, when we did produce this result, this was quite unloving especially for people who have just come out from the first course in fluid mechanics, where they have been told time and again this importance of similarity parameters. And here, we are saying that is not so. What it actually means that there may not be a single similarity parameter that one should be looking at. In this case, of course, the excitation at the relevant frequency, that is, frequency spectrum should also be an independent variable in the study. And, if we do not do it, we are going to be buffered like this. So, that is why I would say that please look at our results once again in a different perspective.

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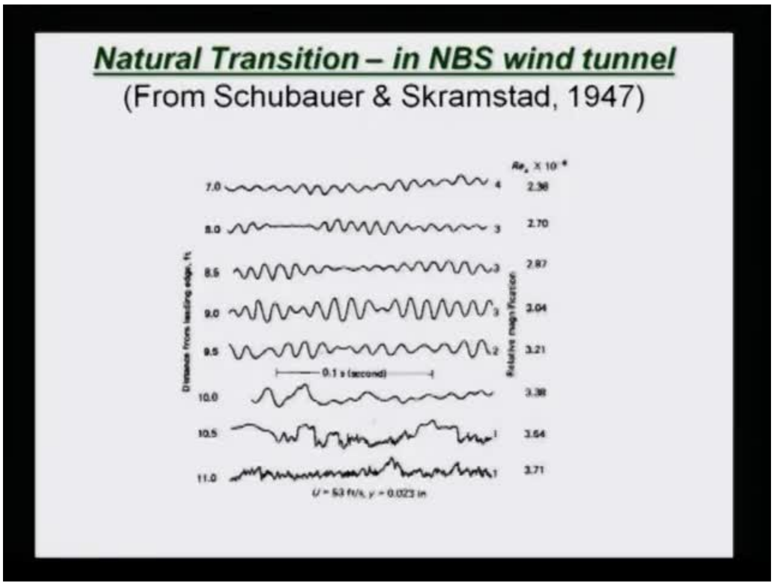

We need to really have intimate knowledge of the input disturbance. And, this is what we saw that in actual flow, even for a flow past a flat plate, Schubauer and Skramstad did show that natural disturbances are not as sort of coherent as one would like to see in a well-designed experiment or in the theory of instability by normal modes.

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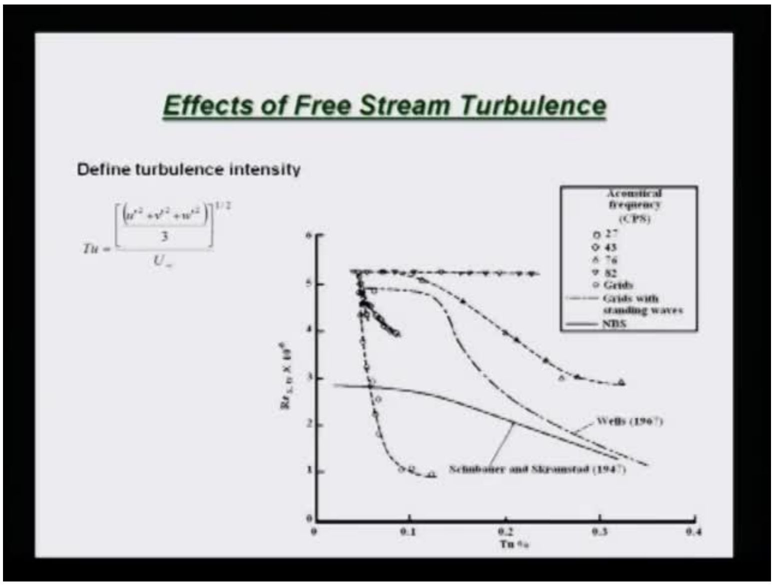

And, this we discussed. Now, you can see so-called effect of free stream turbulence for flow positive flat plate. And, here you have a complete mishmash; you have different kinds of disturbance environment and you have different types of transition Reynolds number.

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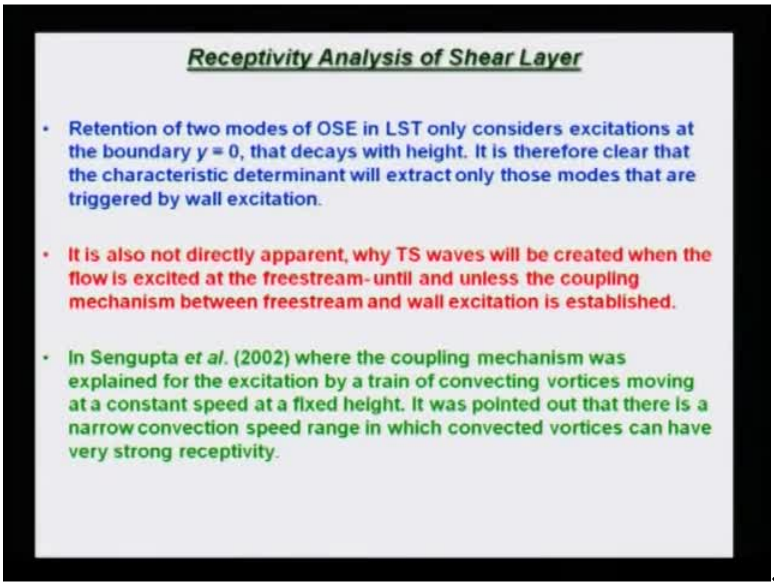

And, this basically is sufficient for us to be convinced that we need to do a receptivity analysis. And, that is where we actually stopped. We said, what do we do when we perform an instability analysis? What we do is basically we try to solve Orr-Sommerfeld equation and we saw it was a fourth order ordinary differential equation. So, it has four modes. Two of those modes decay with height; two of the modes grow with height. And, in formulation and in the analysis, what we did? We only retained those decaying modes. So, what kind of disturbances are we talking about? That any disturbance that will decay with height. So, that cannot be a free stream turbulence effect, because free stream turbulence would be the other way that it may not be able to penetrate inside the shear layer, but it is very much present outside. So, here that kind of a theory can only be verified by an experiment of the kind that was done by Schuhbauer and Skramstad. They excited the flow from inside the boundary layer. You see the connection now. Unfortunately, eigenvalue theory would not tell you that.

And, what we basically then conclude that the stability theory results are very relevant for wall excitation. And, those modes which decay with height, we call them as the wall mode. You are now in a position to appreciate the fact that there would be cases, where you would not be exciting the flow at the wall, instead you would be exciting the free stream. And, if those modes are unstable or they can indirectly excite the wall mode, there has to be some coupling mechanism. You create a disturbance in the free stream; somehow, it gets coupled and the wall modes get excited. Then, you can generate this way actually, (( )) in 1999, published a paper, where he showed the same thing that you vibrate a ribbon in the free stream at that frequency, at that fixed frequency; then, you can see, it gets coupled directly and you get again TS wave at the wall. So, there are ways and means. We will talk about how this coupling is performed.

However, this is the paper that we wrote in 2002, showing how this coupling mechanism works. We looked at a problem of different kind. We talked about a train of vortices going over track plate. We know, the free stream turbulence could be a sort of a combination of this unit process. The unit process we are talking about – either flat plate, I have vortices periodic passing over convicting vortices. So, that is what we did study and (( )). Then, we realized that when we are looking at these vortices moving at a constant speed, that is rather interesting. Have you ever sort of paused and thought about all these things that we are talking about inputs, which are steady, but all instability points add disturbances, which are time varying. So, here also, we are doing the same thing; we are talking about a train of convecting vortices moving at a constant speed at fixed height. At this point, you will see unsteady effects come up down in the book over the plate. So, this actually was performed to find out what was the speed range at which this coupling is maximum. We did it because, Jim Candle of JPL did some experiments in a wind tunnel and he did see a selective band of speed at which this coupling becomes very important. We will talk about that also.

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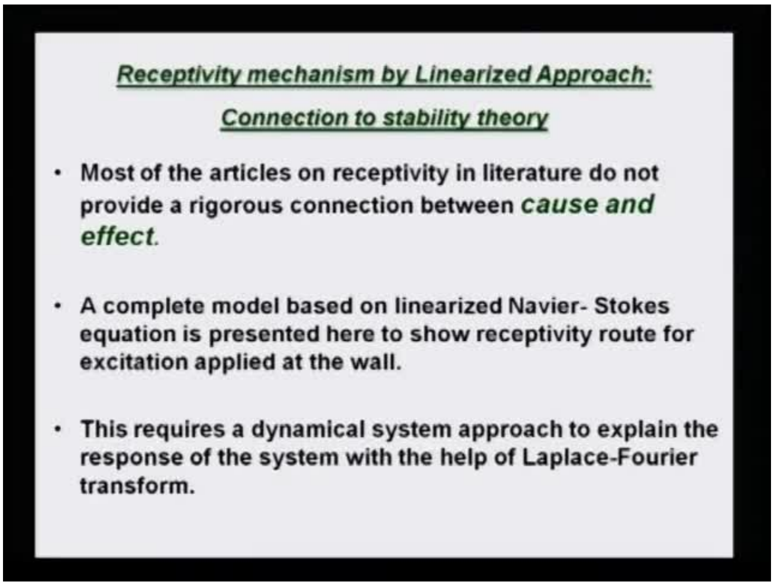

But, let us now talk about how we set up the receptivity problem. We can do many things. We will start off with the simplest possible way of looking at the linearized Navies-Stokes equation and show how receptivity route for excitation applied at the wall creates shears. So, this is essential. This approach of doing problem is called a dynamical system approach. So, basically, we have a fluid dynamical system that is like a black box. You have some input; you are noticing some output. Your task in a dynamical system approach is to relate this input with the output. That is what we do.

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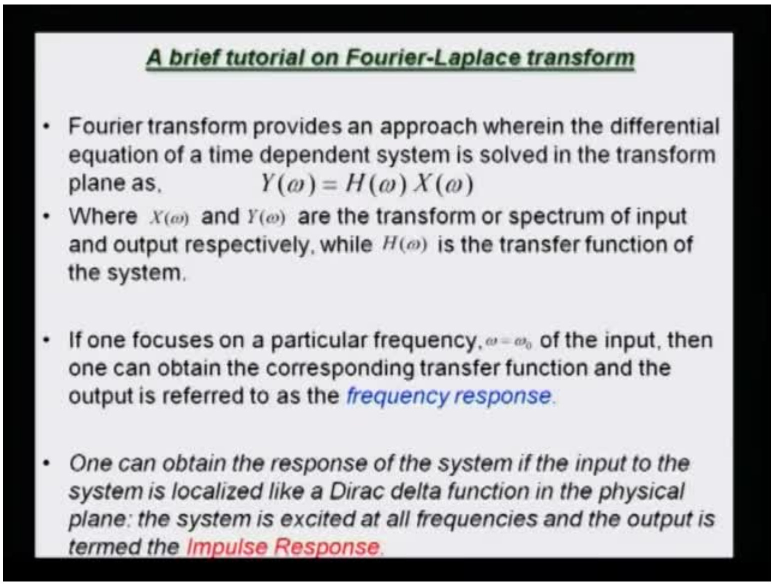

We will make use of some mathematical tool in terms of Fourier-Laplace transform. So, be ready to understand that.

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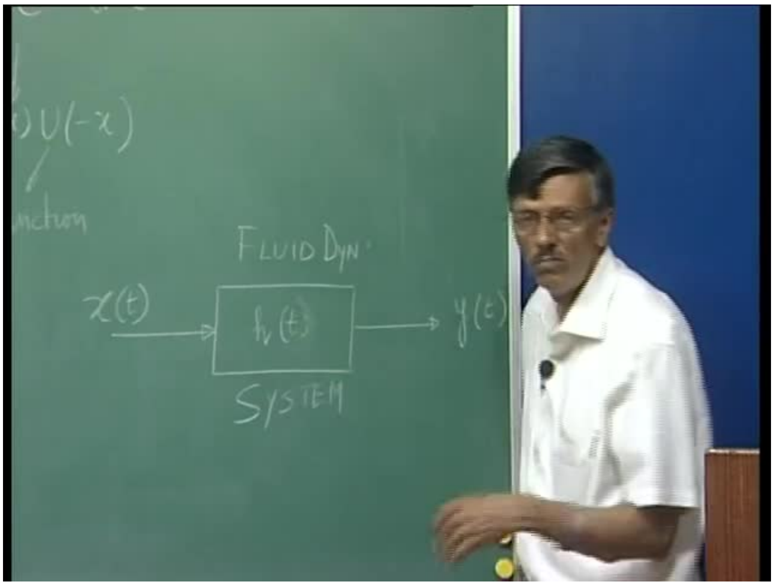

And now, we are talking about dynamical system approach. So, this is the fluid dynamical system and this is basically being excited; let us say some simple thing; I call it x of t. And, the output that I get, let me call that as y of t. So, there is something this black box is doing. That is what we call as the transfer function. I will call that as h of t; h of t and then it will be function of all the associated parameters of the problem like what we showed for the cylinder. There, Reynolds number is one of the parameter, which was well fixed, but there was the other parameter of the various time scales; and, the time scale means in that case, it was the amplitude at strouhal frequency; was a thing. So, we can have all kinds of parameters here, which we can call p, q, etcetera and then we get this. What happens in Fourier-Laplace transform theory or the Fourier-Transform theory itself what we would show, (Refer Slide Time: 17:51) that the output is nothing but product of this transfer function in the omega space multiplied by the input spectrum. So, output spectrum is a convolution of the input spectrum and the transfer function of the system.

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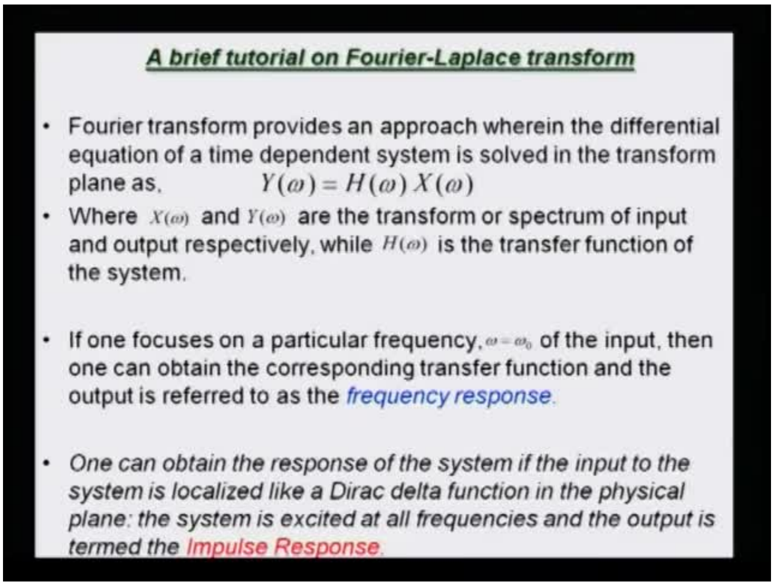

Now, there are two ways of looking at this. We have done some experiments in the lab, etcetera. In dynamical systems or vibration analysis, what you do? You have done in a structure dynamics course. You vibrate the system at a fixed frequency; you have a shaker and then you excite the structure with the shaker; and then, you measure the response. That is what is called as the frequency response, because you are focusing upon a particular frequency, the frequency of the shaker. That is what we call as the frequency response.

There is the other experiment that you have done even earlier in your school. When you did that experiment of seeing of a pendulum, what you did? You just gave a disturbance at t equal to 0 and let go the system. That is what we call as the impulse response of the system. You just give an impulse and let it go; whereas, here in frequency response, you are continuously exciting the system at the same frequency. So, these two are different. But, they are sort of complimentary; the only interesting part is what you can say whether interesting major difference between impulses response and frequency response would be. Frequency response would actually latch on to the frequency of excitation. So, it is like a forced vibration problem; whereas, impulse response – what it does? It allows the system a freedom to choose its so-called natural frequency. So, that is why you should see that there is a merit in distinguishing between the two. Although you can construct one from the other, but be careful in interpreting the results. So, we will talk about this. But, before we do that, let us talk about some ground work that we require.

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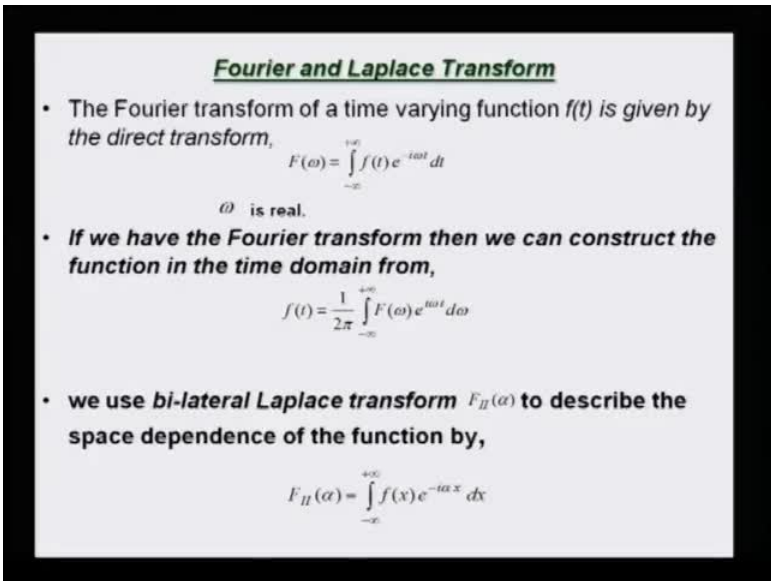

Now, first, talk about Fourier transform that you are familiar with. Fourier transform of a time varying function, f of t, is given by of F of omega. How it is defined? I just simply do e to the power of minus i omega t times f of t and perform integral. What happens in this Fourier transform, omega is real; omega is always considered real. Please note that I have given the range of time from minus infinity to plus infinity. What does it mean actually? What does time t equal to 0 mean? When we are talking casually about instability theory, we do not even comprehend that. But, when we are talking about dynamical system theory, then the time origin makes sense. This is where you begin the experiment. So, if I want to study a system, I switch on the system at sometime. That would be t equal to 0. So, that is why, although in notational convenience, people do right from minus infinity to plus infinity, you would probably writing about 0 to infinity. That would be a better thing to do.

There is also associated issue of what we already know, is a causality principle. I cannot expect to see some effect for something that has happened before even I started the experiment; it does not make sense. So, causality actually precludes that you consider this range of integral from minus infinity to 0. You would be well advised to start it from 0 to plus infinity. Now, if we have this quantity – Fourier transform, this is called the direct transform; as a function of different omega, I can synthesis my signal also like this (Refer Slide Time: 22:23). So, basically, when you do frequency response, what you do is, you try to get this F of omega. And, if you do it over a large range of omega, and then, you can construct the corresponding time varying function, which will have a mixture of all these frequencies acting together. So, that is one way of looking at it. Now, that is about time. Time is special; causality is special.

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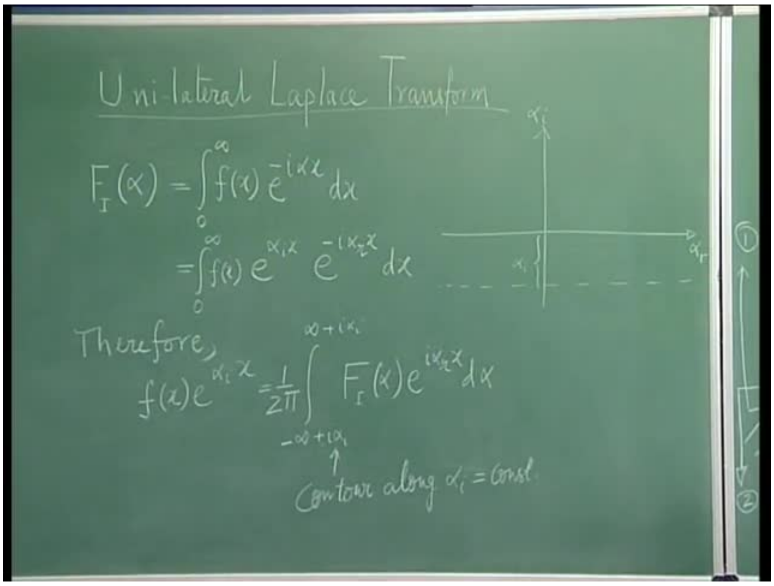

When you look at space, then you do make use of what is called as a Laplace transform. Laplace transform basically is the counter part of Fourier transform, but now, the independent variable is space. And, when you do about the space, we can do about what we just now talked about in the Fourier transform. I could just talk about a unilateral Laplace transform. Why it is unilateral, because the range is from 0 to infinity. We are excluding minus infinity to 0 path; that is why, it is unilateral; we indicate it by subscript I. So, this is given by this. And, what is important for us to realize that this part – earlier, omega was real. But, now, we are talking about omega as complex, which has a real part and has an imaginary part. So, what happens is, I can see that the real part constitutes the phase; is not it? This is going to be your combination of sin and cos. So, this is the phase path; whereas, this path actually directly conspires with the function to alter the amplitude itself, the magnitude itself. So, this is something what you need to do.

Why do we need to make this alpha complex? There are some nice elaborate theory, but we can understand it very clearly that existence of a Fourier transform is not always guaranteed. But, suppose I take f of x, I can measure f of x and then I can multiply by this factor, (Refer Slide Time: 24:52) which will attenuate that. Then, the integral may be convergent. So, if I take alpha i negative, then what will happen that the original problem, if I just taken a real alpha r, maybe difficult to perform this integral. But, the moment I multiply it by e to the power alpha i x, then what happens? As x increases, those contributions are going to be done, because alpha i is negative. So, this is how we can conceive off the requirement of making alpha complex that we may or may not have the Fourier transform. By resorting to Laplace transform, we can improve its convergence property, so that it exists. So, this is what we need to do.

Now, what we can do is suppose in the alpha r alpha i plane, if I look at, what I could do is, I could take a line, along which I will perform this integral (Refer Slide Time: 26:12). That is what I am doing; I am taking a line, which is say, alpha i distance from the real axis. And, I am performing the integral from minus infinity to plus infinity, but along a line for which alpha i remains constant. So, this is this (Refer Slide Time: 26:37). This type of contours are called Bromwich contour. So, we take a particular contour to evaluate this integral and we are talking about that as a Bromwich contour. And, in this case, the Bromwich contour has been chosen in such a way that alpha i remains constant. If alpha i remains constant, now, what I can do? Now, I can perform the inverse transform, inverse Fourier transform; on which function now? This multiplied by this; (Refer Slide Time: 27:12) that is what I have written here; f of x e to the power alpha x is given in terms of this quantity. So, now you can see, this is the direct transform; this is the inverse transform; that is, for the unilateral Laplace transform. And, what we did? We purposely choose a value of alpha i negative. So, what happens is, if I take alpha i further negative, then what happens? If that particular alpha I, the integral is convergent, if I reduce alpha i further, it will be further more convergent. So, that is one thing that we can do.

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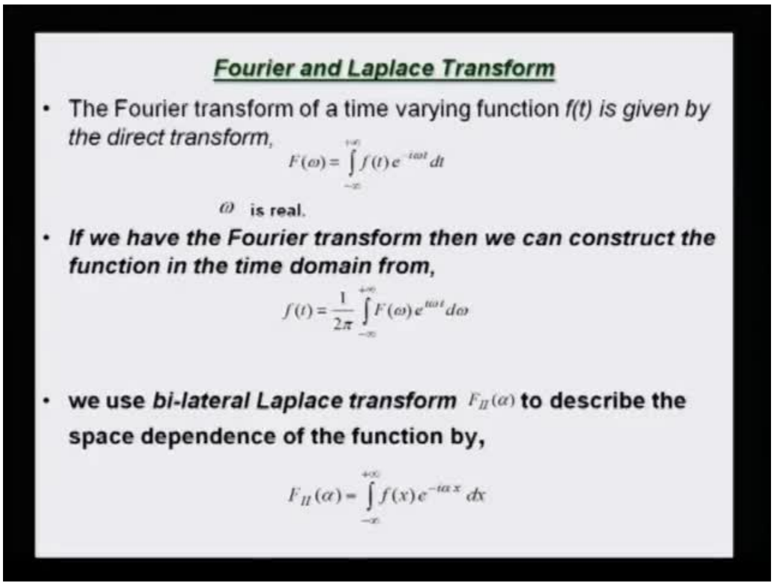

So, now, what we can talk about, as I said for space variation, which are not constrained by causality. So, I could actually take x going from minus infinity to plus infinity. That is what we are doing. When we do that, we call that as bilateral transform and that is indicated by subscript I I 2. So, F 2 of alpha – again, I perform that integral going from minus infinity to plus infinity for all possible (( )). So, that is what we like to do.

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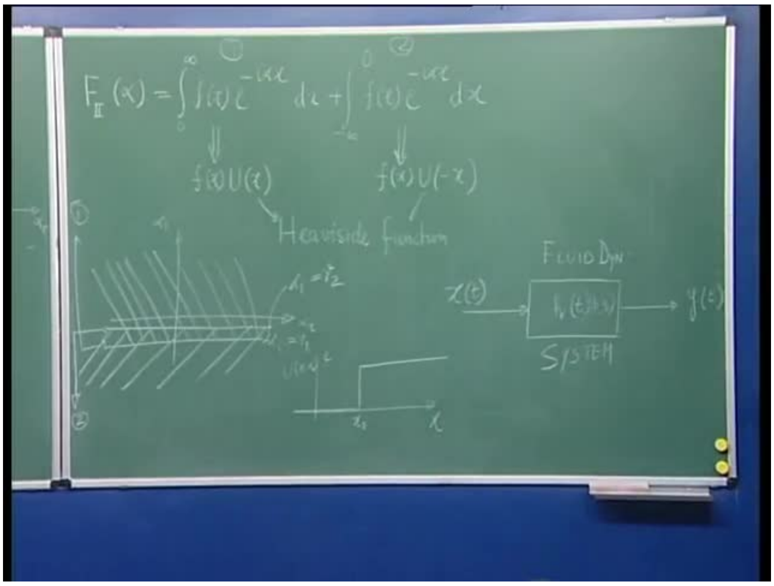

Now, if that is what we adopt, then we can write it in that as a sum of two quantities. This path range goes from 0 to infinity and this path goes from minus infinity to plus infinity. So, if take f of x, which could take all possible values, this is nothing but f of x times U of x. What is U of x? It is a Heaviside function. What is Heaviside function? For all that means, do it like this, say U of x minus x naught. So, what I do is, I mark up a place x naught. And, this has the property that this is 0 all the way up to x naught (Refer Slide Time: 29:30). Then, it goes there and remains constant. This value is 1. Also, it is called unit step function; Heaviside function or the unit step function. So, it takes a unit value and goes like a step. So, that is what it is. So, my original function can take anything. But, because of this, I am only performing integration from 0 to infinity. It is almost like multiplying that f of x into U of x.

And, this one, (Refer Slide Time: 30:00) I can simply think of f of x multiplying U of minus x, because that what it would be; for all negative x, it will be 1; for any positive x, this will become 0. So, that is the complementary thing. So, what happens is, as I told you that if I do this transform and I get that alpha i x there, if that alpha i x for a particular alpha i x, it has worked, what will happen? It will work for any other alpha x. Remember, this was negative (Refer Slide Time: 30:42). So, now, it can go up. So, that is what it is. So, this first integral would be given by the shaded area like this, which is above this line. So, this line, if I call that alpha i is equal to gamma 1, that is the lower limit. The same way, the second part would be defined in a region, which will be below another line. And, this line, (Refer Slide Time: 31:07) I will call it as a alpha i is equal to say, gamma 2. That means what? A part of the integral is valid in this region; another part is valid in this region. So, the total together will be valid in this cross hatched area. So, this is the way you construct the Bromwich counter, which says that it has to be taken from minus infinity to plus infinity. But, you do not have the liberty to take it everywhere. So, by this logic, you can say that this is the region where the Bromwich contour can lie.

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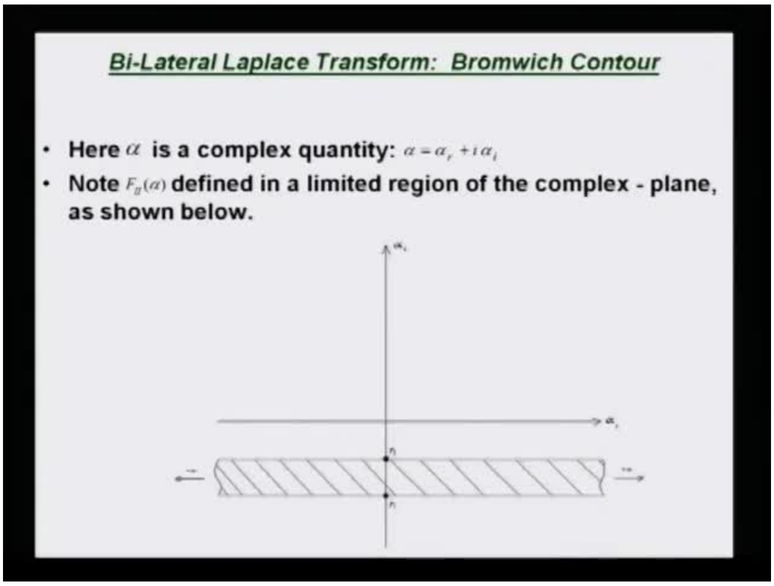

This is something that we keep back of our mind that given a problem, we need to identify a Bromwich contour along a strip of convergence. So, this is the strip of convergence. Now, you can see, we have worked it out. A part is valid in this part; another part is valid in this. So, this is the common region, where both the parts are valid. So, that is the strip of convergence; that is, span by gamma 1 and gamma 2. And so, when we say choose this complex wave number, alpha r, alpha i, we need to take it somewhere. There is no need for you to take it along constant alpha l line. It is only that, you should be in the strip of convergence; you can do anything that you wish. But, you would notice that we do very efficiently when it comes to fast Fourier transform. And, what we do in a fast Fourier transform, we go along the real axis, omega r.

Here also, we can adopt the same picture. I can just shift this thing to this (Refer Slide Time: 33:02). That is what we have done here. Whatever the effect was, we just shifted it by e to the power alpha x and then we performed integral for alpha r ranging from minus infinity to plus infinity. So, I could perform this part very easily using FFTE routines. And, this is what we would be doing, because we would like to use FFT. It is just a matter of importance for us to remember that what we get out of fast Fourier transform is much more accurate than if we would have performed some kind of a numerical approximation. This I jokingly call Slow Fourier transform. If you compare the Standard Fourier transform with fast Fourier transform, it is actually the fast Fourier transform is more accurate. So, do not think of some problem; I will spend more time; I will write out a program and I will evaluate it slowly. That will be not only slow, that will be inaccurate also; so, we should remember this. So, now, we know what a Bromwich contour is. I suppose from this point on, you should not be very much surprise. Unfortunately, though not many people use this Bromwich contour integral method. So, what you are seeing here is what we are going to perform. So, I have underlined that Bromwich contour lies inside that strip of convergence; and, that strip is defined by alpha i ranging between gamma 1 and gamma 2.

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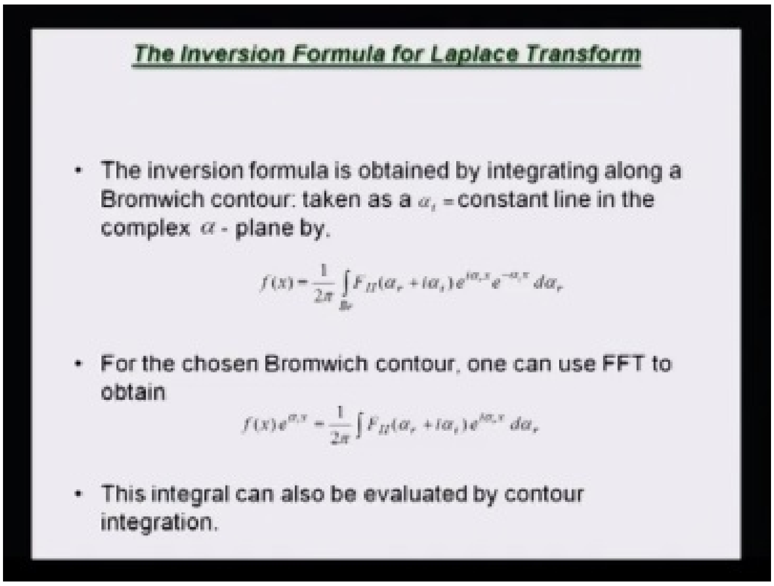

This is what we have worked it out on the black board. So, you have no problem in understanding what we just now talked about.

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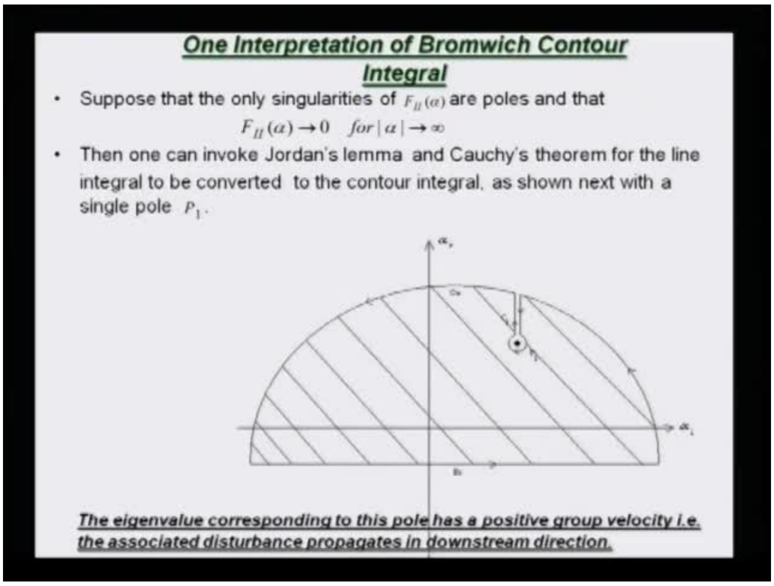

Now, let us talk about the interpretation of Bromwich contour. What does it mean? You see, we found out that one part of the integral is valid for x positive; another part of the integral is valid for… However, what we are doing, if we are inside the strip of convergence and we perform that integral along this let us say, fixed alpha i contour here, then what happens? I am going to get information for both what is happening downstream as well as what is happening upstream. Now, this quantity that we are talking about F 2 of alpha – it is something like in Orr-Sommerfeld equation that phi; you recall that Orr-Sommerfeld equation is an equation for the Laplace transform of stream function or the normal velocity that we have established.

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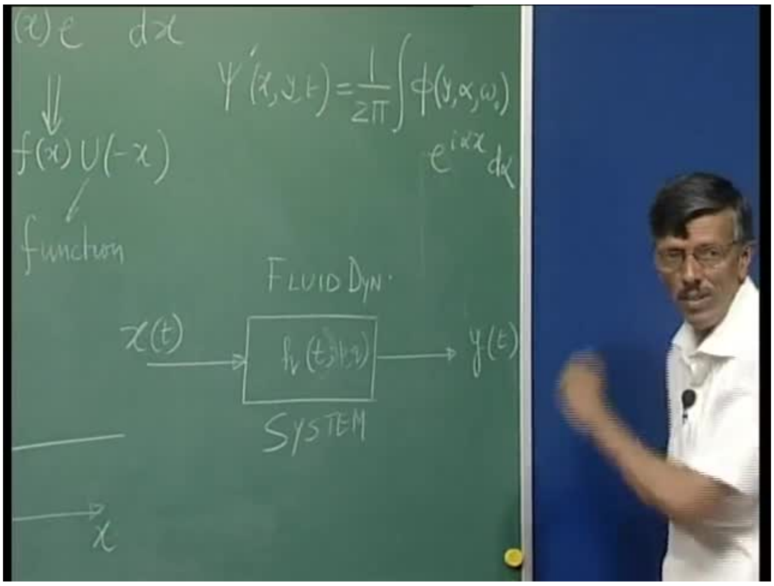

That phi, that equation that we write, is basically what? Disturbance quantity; so, let us say, that is like psi – this thing. And, what we did? We wrote it like there will be factors. So, let us say we make a point of view is this; we write it like phi; that is what we did. And, phi was a function of y; height from the wall. It was also a function of alpha. And, if we excited the system at a fixed frequency, then that would be that; and then, we did it like this – e to the power i alpha x d alpha. So, now, we can see that connection.

This quantity (Refer Slide Time: 37:08) is the Fourier Laplace transform of the disturbance psi; and, the governing equation we have worked it out. So, the governing equation – how did we work it out? It is a ODE; we will solve it. But, when we applied the boundary condition, how did it appear? If we recall, we only looked at the wall mode. When we looked at the wall mode, we removed two modes, which actually grow with y; we only retained those ones, which… Then, we fixed those multiplicative constant by satisfying the boundary condition at the wall. And, in the process, the expression of phi came out to be something like a numerator by denominator. We will do that shortly and we will see what I am saying. But, let me just verbalize what we are going to see. So, this phi itself, when it is the solution of the differential equation and it has satisfied the boundary condition, it takes a form numerator by denominator. Now, what happens? If you recall the exposure to complex analysis that you would have situation that the denominator is 0, and those points are called what? The poles. So, now you can see, all that you have learnt have not gone; on use.

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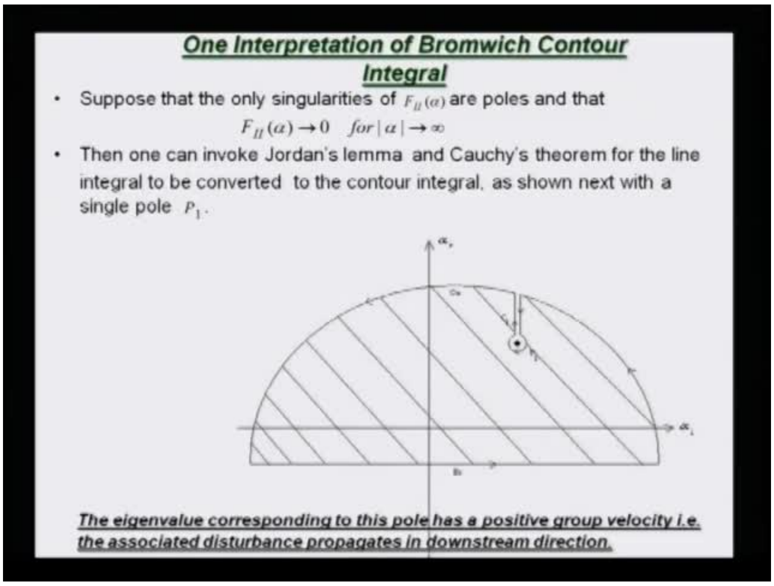

Now, we are seeing the merit of learning all those. And, there also, we did all those integrals going from minus infinity to plus infinity, indefinite integrals; that is what we are going to do. And, this is very familiar to you. You recall that you used to do some integral; if we have to do it from minus infinity to plus infinity, we take a semicircular arc and then everything was taken care of. What we just now said – this F has a role like what we talked about the amplitude of the disturbance function or the normal velocity. And then, we said, it will be of numerator by denominator form and the zero of the denominator provides you the poles. And, those poles are what? They are eigenvalues.

Basically, what does the transfer function now tell us? Transfer function is something like the amplifier. So, if you give a very imperceptible small disturbance and at that disturbance happen to be the pole of the transfer function, then you are dividing by 0; so, you are basically amplifying it. That is this connection between your receptivity and eigenvalue analysis. Now, you can see that what we assumed that we are dividing by 0 means, we are trying to excite the system at those eigenvalues. And, if I have also numerator, it is not exactly equal to 0; or, even if it is vanishingly small, then we have indeterminate form, 0 by 0. So, despite the fact that input is virtually absent, because of this indeterminate form, we still get a finite limit and that is the eigenvalue; eigenfunctions. Now, you can see the connection. However, those values that we will obtain as we have seen, could be a point like this, which will have (Refer Slide Time: 40:40) a value of alpha r and a value of alpha i.

Now, we have talked about group velocity. So, what is the utility of that group velocity that comes to your rescue here? You have identified an eigenvalue, but how do you know whether this eigenvalue corresponds to disturbance that is going down steam or upstream? You will have to find out the associated energy is going in which direction? And, that direction of the energy propagation is given by the group velocity. So, let say this point that P 1 that we have found out; that is a pole, has a positive group velocity. So, what does it mean? This disturbance will go downstream; positive group velocity. If it was negative, then it will be like this (Refer Slide Time: 41:31). So, what happens is, you must be already started thinking that a prior even when I start a problem, how do I fix my Bromwich contour. So we are giving you the recipe step by step that you identify all the eigenvalues; calculate their group velocities; and then, you see which is going in which direction. So, each of the eigenvalues has to be associated with upstream propagation or downstream propagation. So, in this case, (Refer Slide Time: 42:05) P1 corresponds to the downstream propagations.

Then, what happens is, I am performing the Bromwich contour integral from minus infinity to plus infinity like this. Now, what do I do? I can close it like this. And, because this is a pole, I will have to isolate it. I dent the contour, indent the contour, and go around. Then, what happens? Inside the shaded area, the function is analytic. Then, you can use all of those nice properties of Cauchy's theorem, Cauchy's integral formula to help you in finding out the response. However, you realize also, in all such activities that you have done in a complex analysis course, you have assumed that the contribution of the integral along this semicircular arc is 0. That is what is called as the Jordan's lemma.

Now, you see, Jordan's lemma actually gives us a very vital guideline. How do we use the Jordan's lemma, which says that the integral along this is going to be 0? So, what happens? If I perform this integral and this is a closed integral, the function inside is analytic, what happens to the integral by Cauchy's theorem? 0. So, it is 0. So, what happens is, this integral plus this integral (Refer Slide Time: 43:42) equal to 0. But, then, what have we done? We have to also include the contribution coming from there, so that contribution goes to the right-hand side. So, what happens is, then, integral along this Bromwich contour in the presence of Jordan's lemma is equivalent to 2 pi i times sum of the residues; that is the Cauchy's theorem. So, that is how we go about it. It is all very nice and clear, but the only thing is you will have to make sure that Jordan's lemma is good enough here.

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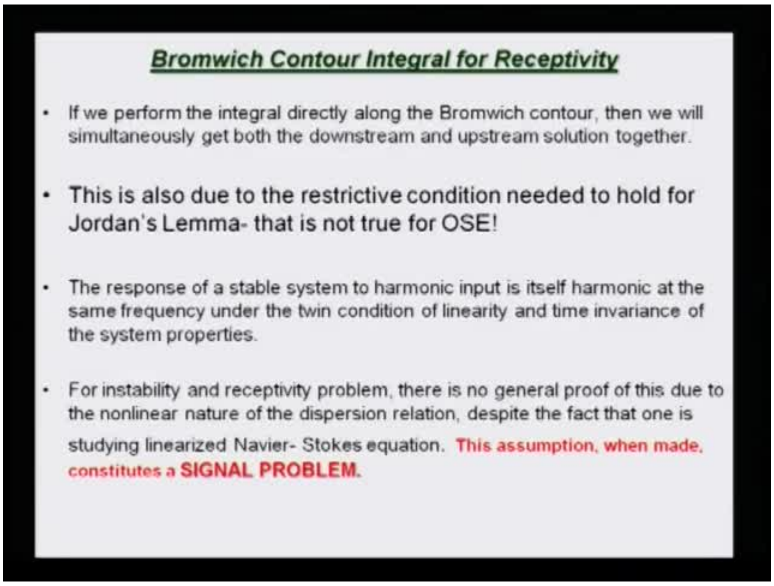

And, there are some ways of finding it out whether the Jordan's lemma is good or not; we can figure it out. But, if Jordan's lemma is good, then if we perform the integral directly along the Bromwich contour, then we are going to get the solution for both upstream propagation as well as downstream propagation. You see, somehow if you look at all the text books on instability theory and the associated mathematical formulation, they just simply have no clue; they just simply say, up to some distance, there is nothing, all of a sudden disturbance comes and it goes in the downstream direction, and so on and so forth for a fluid flow.

Now, if that happens, then we are talking about some kind of a discontinuous behavior at the excitation. But, we are talking about incompressible flow. How can you reconcile to that fact that you have very analytic functions; solutions are supposed to be possessing all kinds of derivatives and continuous. And, at the same time, at the excitation, you have a job discontinuity. This cannot be necessarily true. So, what happens is, this Bromwich contour integral solve that issue, because it gives a solution both upstream as well as downstream. And, what is more important also you realize that instability theory; what we did find out? We did look at the solution contribution coming from individual eigenvalues one at a time. And then, that is what we called as a normal mode analysis. So, each of the individual eigenvalues is one of the modes and we sum it over; that is what we do; 2 pi i times sum of the residues. So, each one of them working independently; but, we have also talked about the group velocity. What did we see? That waves interact and it can form groups. But, this normal mode analysis does not seem to give you a direct connection to that. So, we have to be careful in interpreting our results. Furthermore, we would also have to ensure that the solution of Orr-Sommerfeld equation has obtained satisfies Jordan's lemma. Now, this is one issue.

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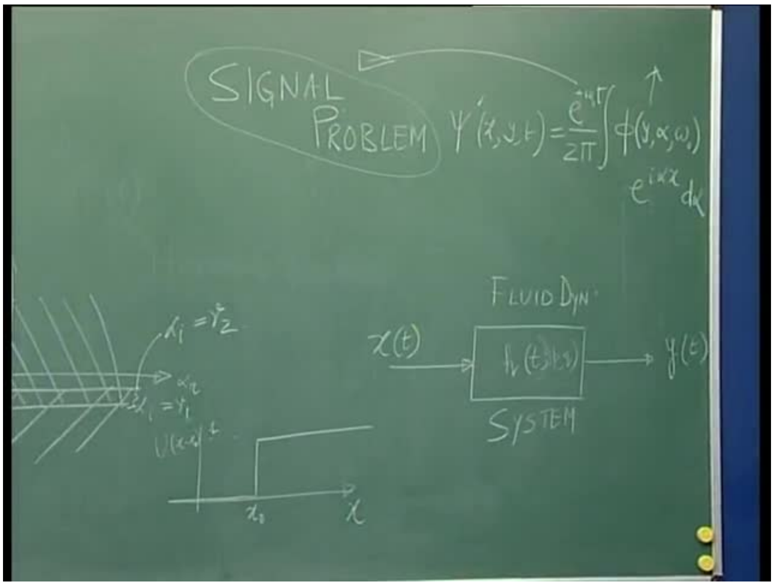

We have also made one assumption here; as you can see here that we have performed the integral over alpha. What did we assume? That here, the system is excited at a fixed frequency omega naught. It is an assumption. If I excite a system at a frequency omega naught, for a stable system, I can think of that response also will be at omega naught. But, what we are doing it for alpha allowing the system to pick out its natural frequency can happen also with the time variation. So, what is the guarantee that if the input is at frequency at omega naught, output also will be at omega naught? This is more like an assumption. Whenever we do that, we call that as the signal problem. So, if I excite a system at a frequency omega naught, output is also at omega naught. This is what we call as the signal problem. So, please do understand that this is an assumption, which we need to really check it out. You can very clearly see how to check it out; that you do not need to make that assumption; you can also perform the integral over omega. Whatever you are doing in alpha plane, you can also do it in the omega plane. And now, omega also is going to be complex. And, you will have to be doing this Bromwich contour integral in the alpha plane, in the omega plane; and then, you would get a composite solution. We will talk about some of those exercises.

However, before we do that, we need to understand that what we have achieved so far; that we are studying linearized Navier Stokes equation. By now, all of you are with me that Orr-Sommerfeld equation is nothing but the spectral representation of linearized Navier Stokes equation. I have just written it in the alpha plane; that is what we have done. So, we need to really do some of the studies that we need to be aware of.

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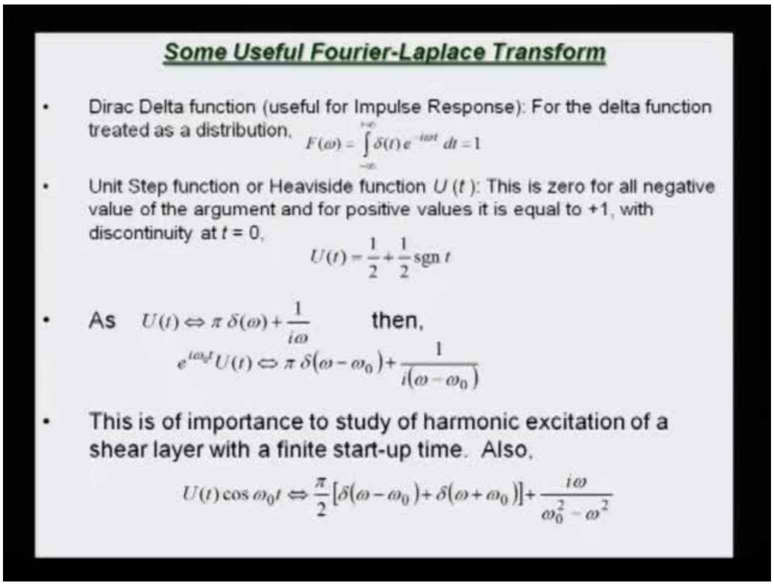

Now, let me talk about some simple Fourier-Laplace transforms, which are rather useful. However, I think before I do that, I should tell you little bit about properties of Fourier-Laplace transform, because you may not all have done it before. So, basically, we will talk about some properties. So, it is basically a short tutorial on Fourier-Laplace transform. So, let me go through it simply with you.

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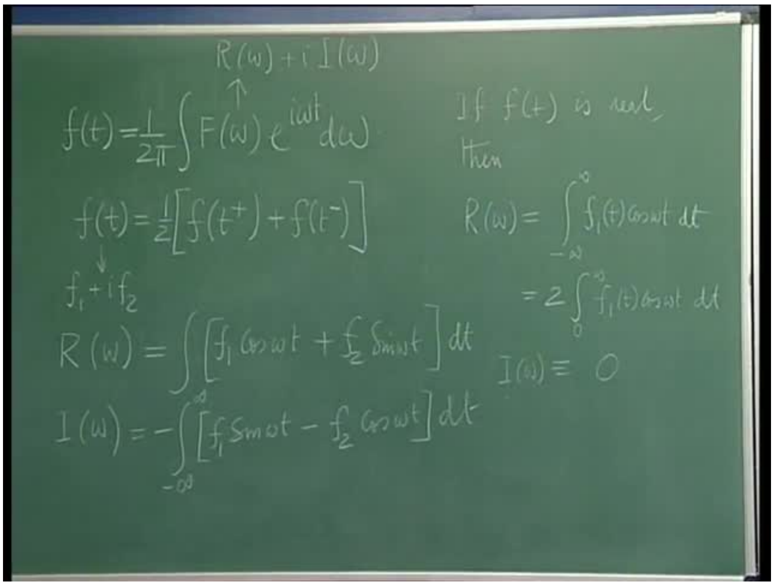

Then, say, if I talk about a function f of t, I said from the… This is the inverse transform. So, what we do is, we take this direct transform, F of omega and perform this integral e to the power i omega t d omega. Now, at any point of discontinuity, wherever we may have as we have seen; like unit step function is discontinuity. What we do at the point of discontinuity? That could be written as the average of the right-hand limit and the left-hand limit of the point of discontinuity. So, that is the usual way that we should be talking about.

Now, f itself could be let us say, f 1 plus i f 2. Now, what I could also do is I could also represent this F of omega (Refer Slide Time: 52:03) as real part plus an imaginary part. This is what we should have. So, what one can show rather clearly that R of omega, the real part of the Fourier transform can be obtained from this f 1 cos omega t plus f 2 sin omega t dt. I just simply opened up the integral line and then we performed this integral, that is, R of omega. And, the imaginary part will be minus of this; (Refer Slide Time: 52:55) that will be f 1 sin omega t minus f 2 cos omega t dt. So, we can obtain the Fourier transform.

See basically, you can understand that we are not talking about physical system. So, f of t is kept in a general form; it has a real and imaginary part. For a real function of course, you will not have that luxury. What happen is, if f of t is real, then we can very clearly see that f 2 is 0. So, R of omega would be nothing but f 1 t cos omega t dt. And, this integral is of course, from minus infinity to plus infinity. Cosine function is what? Is a even function or odd function? It is an even function. So, what I could do is, I just simple write it as two times 0 to infinity f 1 t cos omega t dt. So, whenever you can write it like this, (Refer Slide Time: 54:41) you call this as a cosine transform. The same way, one can obtain the value of i of omega. What happens to i of omega? Of course, this part is gone and this is a odd function and I am performing integral from minus infinity to plus infinity. So, the negative part will cancel the positive part. This will identically be equal to 0. So, we can make use of that property. Now, this is of course, true when we have said this f 1 t (Refer Slide Time: 55:32) is real and even; we have to be careful. If f of t itself is real and odd, then all these logic flips over; then, R of omega will be 0 and I of omega will be told two times the split integral. So, please do understand some of these subtle issues.

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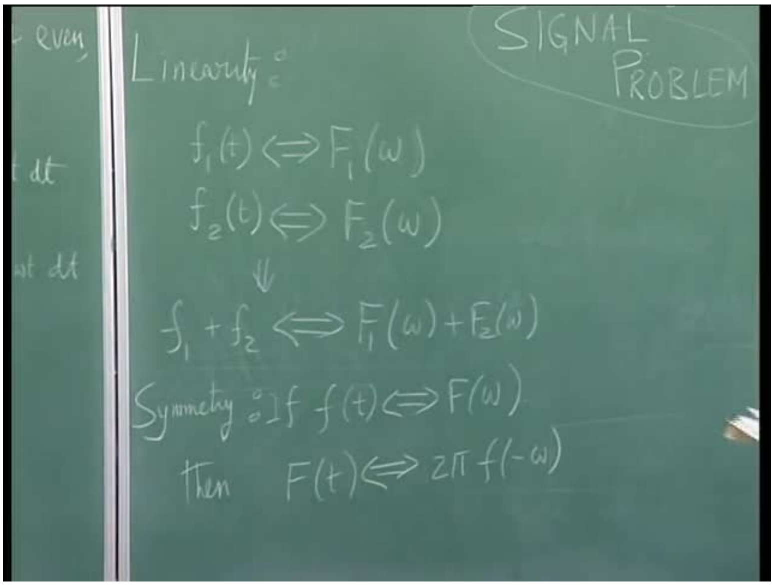

Now, let us enunciate some of the properties of Fourier transform. One of the properties is the property of linearity. What does the linearity property imply? That if I have a function f 1 t, whose Fourier transform let us call that as capital F 1 of omega, and similarly, if I have another function f of 2, whose Fourier transform let us say, it is given by F 2 of omega, then linearity simple says that f 1 plus f 2 is the Fourier transform of F 1 plus capital F 2. So, this is a linearity property. Please do understand that it is not necessary that your actual unknown have to be equal to linear function; it could be non-linear, but this transform is a linear thing. So, it can be applied to (( )); irrespective of whether the function is linear or non-linear, it does work out the same way.

Now, there are some other properties, which are quite often used. For example, one, often uses the symmetric property. The symmetric property says that if I have a function f of t related to its Fourier transform f of omega, then I could show very easily, this F of t is a Fourier transform of 2 pi times f of minus omega. I am just enunciating; you can actually plug it in the basic definition and can show it. So, that is the symmetric property.

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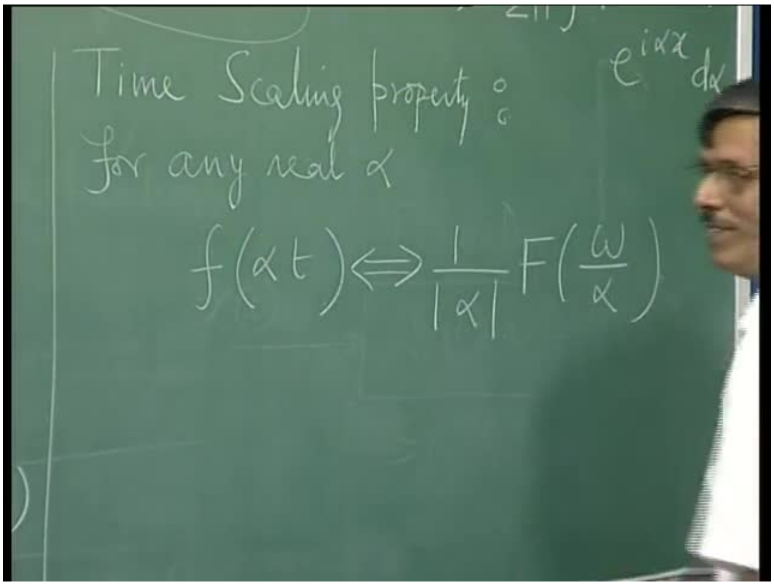

There is something called the time scaling property. The time scaling property comes like this that if I have a real constant let us say, alpha, then I have let us say, f of t given; I scale that time by this function alpha. It is a Fourier transform, would be given by this. So, this time scaling property actually is a very interesting thing. What it says? That if you have a function, which is stretched in the time plane, in the frequency plane, it is squeezed. So, that is what we already know from the properties of Fourier transform. If I have say periodic function going from minus infinity to plus infinity in the time plane, the corresponding Fourier transform is just a single point; it is a delta function. So, same thing happens; it is the other way round. If I create an impulse, I create a disturbance at t equal to 0, it excites all frequencies.

Now, you know that what we were talking earlier about the frequency response viz-a-viz the impulse response; why impulse response is so much more preferable? In performing the impulse response, I am actually exciting all frequencies and the system can latch on to its natural frequency. But, in performing an experiment with frequency response in point of view, I have to really go scan and I will have to worry about if I am missing exactly the peak or not. So, you see, impulse response is a much better thing to do.

I think we will stop.