Instability and Transition of Fluid Flows

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Module No # 01
Lecture No # 11

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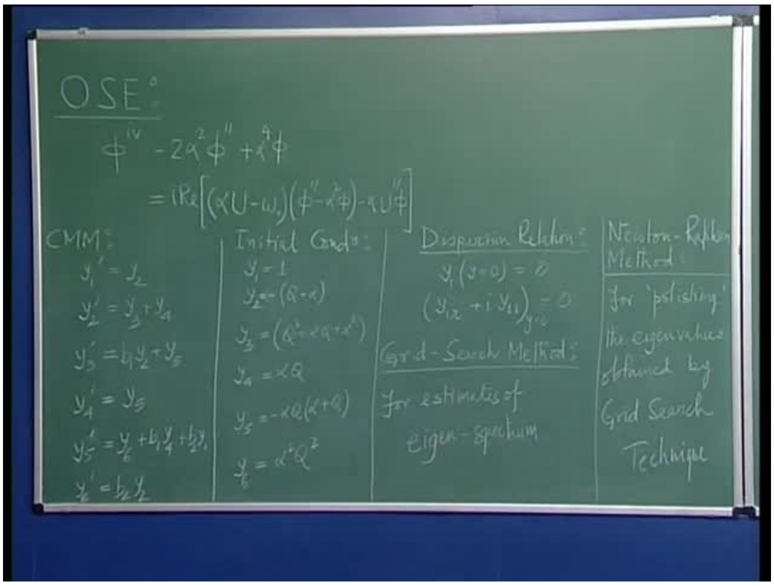

Let us now get back to the discussion in stability study of a parallel boundary layer. Here, in a nutshell we can follow the step; this is our governing equation for the disturbance quantities given in terms of the amplitude of the normal velocity.

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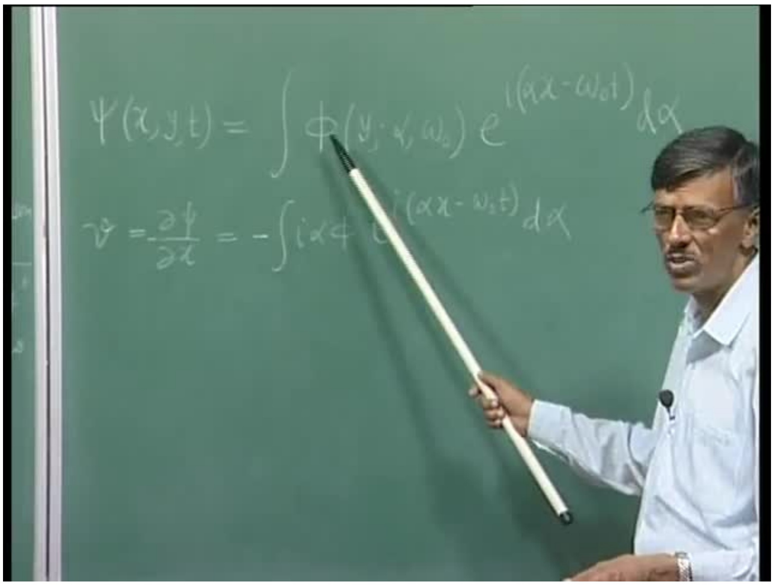

You also realized that normal velocity could also represent the amplitude of the stream function because if I take a stream function like this, suppose I represent a disturbance stream function like this, in terms of phi y say alpha omega naught and into the power i alpha x minus omega naught t d alpha, if you do this then you can see that the v velocity that we are talking about there will be del psi by del x with a negative sign, and that will give you minus i alpha phi e to the power i alpha x minus omega naught t d alpha.

So, you can see whether you ascribe phi with the v velocity amplitude directly or the amplitude of the stream function; it is essentially the same thing. So, that is what we need to keep in mind, and what we did was that we noted this was a Steve differential equation, we converted them into the six set of first order coupled order differential equation for the compartmental variable y 1, y 2 to y 6. And these variables have the property that they satisfy the fast stream condition, so, that is what constitutes initial condition, we have normalized them with respect to y1 and we get this sixth condition.

So, what you do is you start solving these six conditions with this initial condition and come up the wall and investigate if y 1 is equal to 0 or not, that is what we called as dispersal relation, so that will have a real part and imaginary part. Then we talk about a grid-search method, in that grid-search method what we do is, we look at the 0 contours of y 1 real and y 1 imaginary, wherever they intersect they gives us Eigen values, the collection of this Eigen values are what are called as the Eigen-spectrum.

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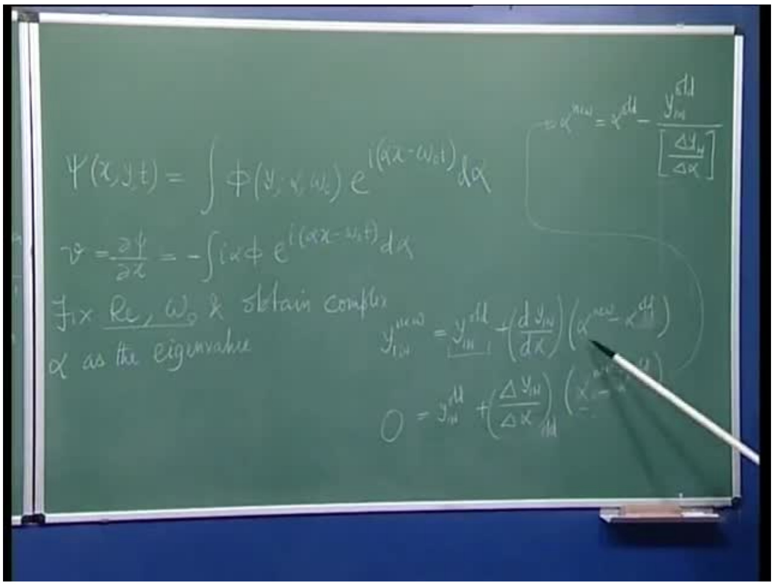

So, once we have the Eigen-spectrum we have to do something more, and what that something more is basically we need to polished this Eigen values better because this gives you a kind of a rough estimate, this is a rough estimate of the Eigen values. How to do Newton-Raphson search, how do you do, well that is what let us spend a little time talking about Newton-Raphson search. First of all we note that we are in search of Eigen values which correspondence to the spatial instability problem, so what to do is we fix Re; the Reynolds number, and omega naught and obtain complex alpha as the Eigen value.

So, what we are going to do is basically, we go through this exercise that we talked about, so these two are fixed; Re and omega naught is fixed. We start with some kind of initial guess of alpha, march down up to the wall, and investigate what is the status, if it is not 0 then, it is not Eigen value, and I told you that this is the way we find it out. So that gives you a kind of initial guess for the Eigen values, then what you do is you do what is done in shooting method, we start off with that value alpha, again from free stream we come and we check for y 1 at the wall. Now, in Newton-Raphson search we will be little more demanding, we will be looking for the desired precession, we will say look the zero that we are talking about it should be 0 say up to the 16 decimal place accuracy and so on so forth. Once we do that we see that it is not satisfying, and then what you do is you basically want to find out y 1 which I will call as the new value in terms of, so this is at the wall, so this what we will say is y 1 at the wall but we obtained as this plus I could obtain d y 1 wall by d alpha into alpha new minus alpha old.

So what we are talking about we have this imprecise old value, so we want to put this value to be equal to 0. If I do that this will be 0, this one will be this plus this I will have a numerical estimate; numerical estimate will give me some kind of estimate of this kind, so this I would be valuating what corresponding to the old condition, so this is basically corresponding to the old condition times del alpha new alpha new minus alpha old. So what do we get from here, we do get this value, what this value is going to be, I will first take this on this side, divide by this and then add to that, so I will find alpha new should be equal to alpha old minus y 1 old divided by this quantity; delta y 1 w delta by delta alpha. So, this is your Newton-Raphson search actually.

So, basically it depends on how accurately you evaluate this first derivative, and how these two quantities are with respect to each other. If they are close to each other then the subsequent terms are negligible; that is the whole idea so that you can just simply take this first order representation and get estimate for new and you can keep doing it again and again, that is the essence of Newton-Raphson search and you find it out what you get.

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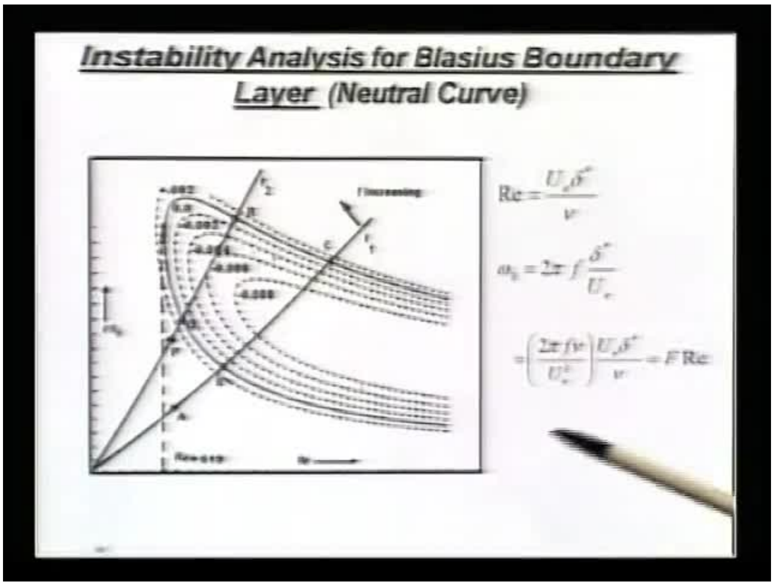

So at the end of that activity you are going to get an estimate for alpha real and alpha imaginary and that is what we like to plot. What I have shown you here is a plot of alpha imaginary in Re omega naught plane. Now this is something what we have talked about in the spatial problem, we fix the frequency omega equal to omega naught. However when we write this, this requires an approximation, what is that approximation that I am exiting the system at a frequency omega naught the response of the system is also omega naught; how good or bad this is. We are looking at linear dynamics, this is linearized perturbation equation, so for a linear system one would intuitively say that this is a pretty good approximation and this is what is called as the signal problem. And the essence of the signal problem is inherent in all published material on stability theory expect some of the work done by us. About couple of decades ago we started thinking fundamentally why this has to be so.

Any good research starts with an innocent question and you should ask that if you want to do something fundamentally research this is the way you should ask. Why did you asked that question that we are looking at linear system and we are exiting the system at a frequency omega naught, why should not put omega naught, why there should be inconsistency, you got to understand it because we all already seen that this is not an o d e. The equation Orr- Sommerfeld equation is o d e why because we are looking in the wave number plane, so that space dependence has been masked there so it is not simply ordinary definitional equation. If it is ordinary definitional equation then giving you omega naught and expecting omega naught in the responsible field is perfectly justified, but we have a space time independence of these problem.

In a space time dependence of the problem we have what we call a dispersion relation. It means so happens there the initial transient at initial times, the dynamics may be more complex than what you are used to expecting from a steady state assumption of signal problem. That assumption relates to what will happen over a long time, but in the initial phases of the evaluation of the variable it is quite unlikely that you will only see omega naught; we will talk about it in greater detail when we do not do such an assumption what do we get. So what happens is in this case we are resuming a signal problem and this is what is the consequence plotted omega naught in the y axis, Re on the x axis; Re is defined in terms of the bound layer velocity, the displacement thickness, and the kinematic viscosity that we are quite familiar with.

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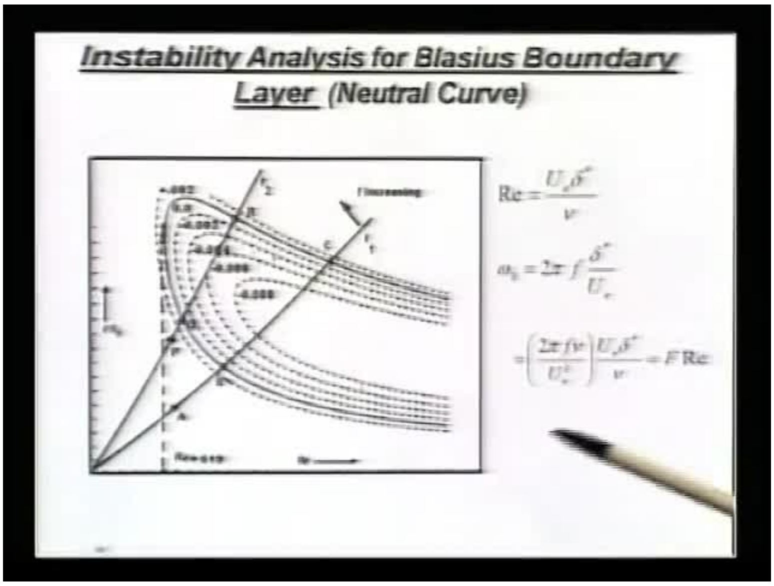

Omega naught is a circular frequency that we can relate to the frequency in hertz by multiplying it with 2 pi, and we known initialize it by multiplying it by a time scale and the time scale is the length scale by the velocity scale, so this is our omega naught 2 pi f (( )) delta star by U e. What we have done here we have reorganize it, then what we have done here we multiplied this 2 pi f with new and divided by U e square so that we have a U e upstairs, and delta star is as it is there, and we have multiplied by nu, divided by nu. So what happens is omega naught is not a product of two quantities f times Re; omega naught by itself is non-dimensional, Re is non-dimensional, f is non-dimensional.

So, what is this f capital F, capital F is the non-dimensional frequency which was stated in hertz that we have converted into a non-dimensional quantity given by the expression within this bracket. So, if I tell you to study the property of a shear layer for a fixed frequency f then what you should be looking at, you would be looking at a constant f value and what is constant f; here f is omega naught by Re, so this is like if I want to keep f equal to constant that has to be a straight line, so that is what we drawn, we have drawn many such straight lines and the straight lines each one indicate a particular value of this f; capital F or lower case f. What happens as the f increases, the frequency increases in hertz we go from here to here, and when we go this way then we see that for each of these combinations we can find out the alpha or alpha i value, so in this whole domain I have a map of what the values of alpha r and what the values of alpha i going to be. Once we look at that we plot those contour lines of alpha i. What we done here the constant values and we noticed a feature of this particular zero pressure gradient boundary layer that there exists a line along which alpha i is equal to zero that is drawn by the solid line, and this line along which alpha i equal to zero corresponds to neutral stability, so that is why this curve is called as the neutral curve.

So what we have done here, we have drawn a neutral curve, what is inside the neutral curve, Neutral curve is the locus of alpha equal to zero; inside alpha i is negative, so that means that region corresponds to instability. Whereas outside the neutral curve you have a stable region. Now you see what happens suppose, I start off from a point a, so at this point a I tried to trap a frequency corresponding to f 1, then what happens this frequency will travels along the straight line all the way up to point b; it is in stable region. So whatever disturbance that I am looking at, it will decay up to b then it will start amplifying, that is what our theory states.

This will amplify and reach c that is again there been the extreme where it is unstable, so be on that again it will be unstable. Now you will also notice the feature of this curve, feature of the curve is that that the instability picks up the rate at which it grows, it becomes more and more as we come inside and then again after some point the contour value start dropping off. So it increases in amplitude in terms of intensity of the instability and then again it decreases, becomes neutrally stable, and then it becomes stable, so this is what we do.

Now, suppose at the same location instead of looking at this frequency f 1 we are looking at a frequency f 2; f 2 is greater than f 1. And then what we notice that from point P to Q again you have a disturbance decaying, then Q to R it will amplify and then again it will start decaying. The shape of this curve is such that we have a line which you could draw it like this which will be tangential to the tip of this neutral curve, what happens to those frequencies on this side, they will never be unstable, and they will be always stable. So this limiting position where a straight line drawn from the origin simply touches this neutral curve defines a Reynolds number below which flow is stable for all frequencies.

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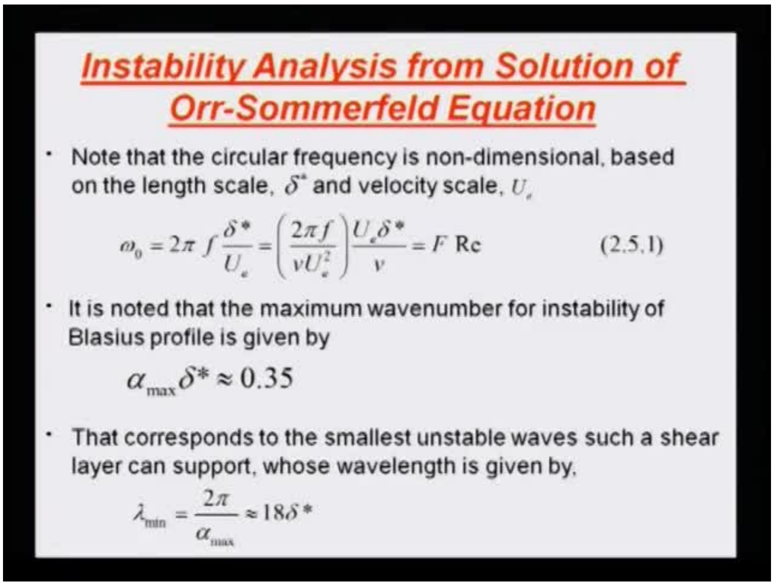

That particular Reynolds number is the critical Reynolds number because it says that below that critical Reynolds number everything is stable according to this parallel flow theory linear theory of spatial instability. So got to remember that this is roughly around five hundred and nineteen for the Blasius profile, and this is what we should keep it mind. And how good or bad this result are we will come to it later But let us now look at some of this other features we have talked about this.

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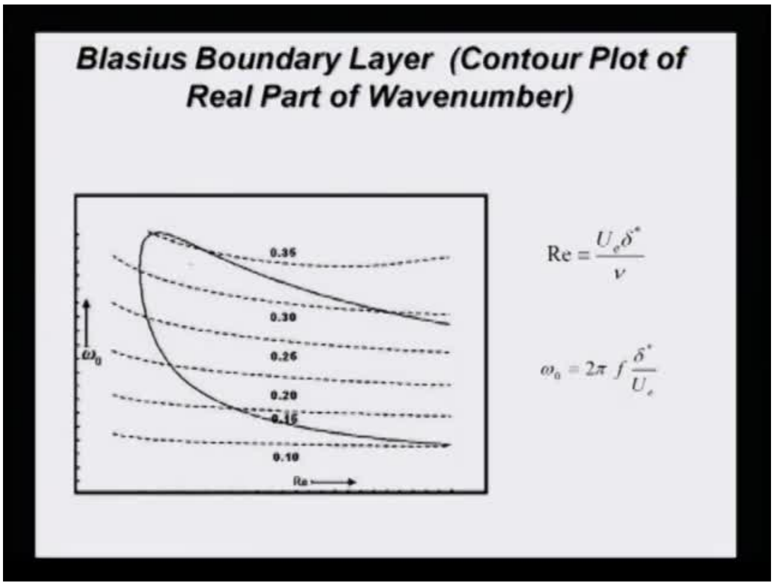

Now you saw that neutral curve was something like this; let me show you something also which is perhaps in next slide, this is what we need to look at. Well we are now looking at the alpha r contours, the real part of alpha contours are again in the same Re omega naught plane and again what we are doing, we are drawing the constant alpha r contours which are like this.

What is this solid line, this is that neutral curve; we had just simply kept it up for our reference purpose. What we notice that if I follow a constant frequency disturbance then what it does, it migrates from one alpha constant value to the next one. If this variation is gentle then what are you going to see in terms of the disturbance profile that it is slowly going to adjust itself to a local condition. This is one of the driving motivations for using the stability theory that we expect the flow to locally adjust itself to the prevalent condition. And that is really bolstered by looking at this gentle curves; they are going very gentle manner, and this was one of the interesting points that people did observe experimentally later that indeed the disturbances do latch on to the local property, and that is what people looked at.

Now let us also look at one particular aspect, the tip of this curve. If I look at the tip of this curve the corresponding alpha r value is roughly around point three five to point three six. What does that mean, alpha r is the wave number that is 2 pi by lambda and this alpha r is increasing, so there is a critical minimum wave number beyond which everything is going to be stable. So that is what we have in the transpose, you have written here that we have a case if alpha is maximum that is roughly, I just simply said it is about value of point three five. Please do understand where all the quantities that we are displaying and we are noticing are non-dimensional.

So if, what is a dimension of alpha, alpha has a dimension of one over length. So essentially what I am writing as non-dimensional alpha in a dimensional form it should be like this, I multiply (( )) length scale. So alpha max is point three five by delta star and that will correspond to what a lambda mean; minimum wavelength that will be 2 pi by lambda max and that is roughly about eighteen delta star. It gives you some idea that in a Blasius zero pressure gradient boundary layer if instabilities are seen you are going to see disturbance waves which are of the order of fifteen to twenty delta star, you cannot have smaller than this; all that disturbance wavelength will have to be about this and above.

So this is something we should remember weI leaved a 2 that the stability theory works fine. I wish the story was that simple, you already know the story, and the theory came quite sometime early on but one of the problems of the theory was that it was virtually not detectable, and the persons who really detected is a group from USA and this is that famous paper by schubauer and skramstad which was published after the second world war. This work was done in and around onset off Second World War, but this was published after the great war.

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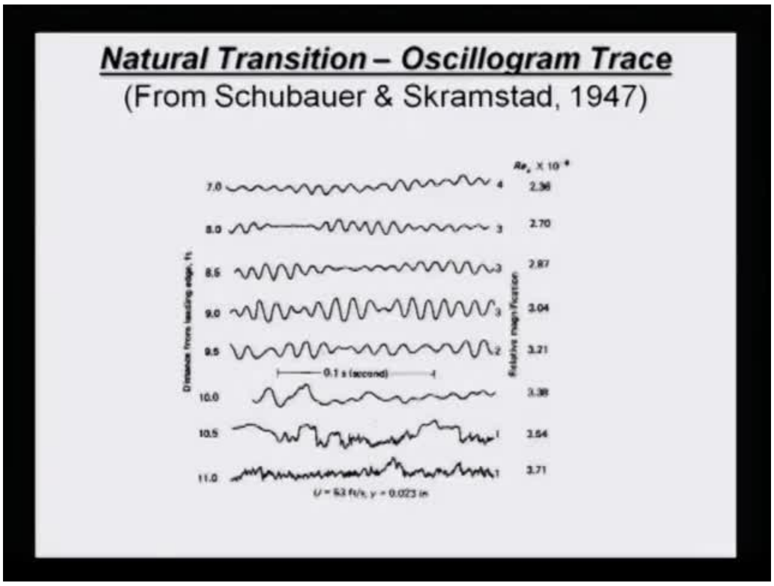

Now what is it that you are seeing here, what you are seeing here is, essentially you take a signal, you put in a probe, what kind of a probe you do, you just put in a hot wire probe inside the boundary layer and put this signal on an oscillogram, and in the oscillogram this is what you are going to see. So these are fixed location, this is about 7 feet from the leading edge of the plate, this is 88 point 5 and so and so, and at each of this location if you construct the corresponding i x Reynolds number based on current length, I can extend to the power minus six scales, and as you seen this is the way this signals look like. So what our theory is saying, well our theory is talked about some frequency amplifying over some distance and decaying outside that range. But, what do we have in an actual scenario that is where we do not impose any explicitly any scale, that scenario will be called natural transition.

So we are doing anything, we are just simply looking at a flow as through waves and this is how it looks like. So in a qualitative manner as you can see there are vary disturbances, but they are quite irregular. I know that is what we talked about this signal problem, but here we do not know how many omega naughts are there, because it is natural, it is not necessarily that you are going to get a monochromatic signal. That is one of the thing we should understand that your response field is going to be polychromatic.

However, despite that what you notice is this following succession, let some you do get a fairly decent signal which looks like a simple harmonic motion somewhere here. However, we notice that this signal what you are seeing here, the trace has been multiplied magnified by a factor of four to see this. So otherwise, if I remove this factor of four this will hardly be seen, this signal is magnified three times, this is magnified three times, three times then, we did not magnify three times, we magnify two times and this is what you get, and then these are not magnified at all. Now, as we can see that starting off from the leading edge as we go along down the flow we see that two things happen, the signal strengthens amplification is seen, and what about the frequency content, frequency content is quite dissimilar, see that is what I have talked about this signal problem. What may happen that initially, I may have a band of frequency, but that band itself keeps changing as we go down, and this is something what you would like to call as most transitional flow or a turbulent flow. So, this is characterized by very high frequency oscillations, this is something we need to understand that two things happen in actual flow, a linear theory; even though it is all right it is going to be valid when the amplitude is smaller because amplitude grows then we see a qualitative change in the flow part.

This, actually you suggested something to those group of scientist in national bureau standard, they realized that if you really want to probe that in stability and transition in a flow you cannot depend upon natural transition, it is a very irreproducible because I could take the same plate and I could go in another tunnel may be in the same lab itself but in another tunnel, and I will do the same thing, I will see the oscillogram trace and I would see something different why because the disturbance background is different in different tunnels. So what one needs to do actually then, one needs to do what we call as the receptivity studies; the receptivity studies imply the following that you try to take out the background disturbance as far as possible, move it out and then you want to test the power of the theory.

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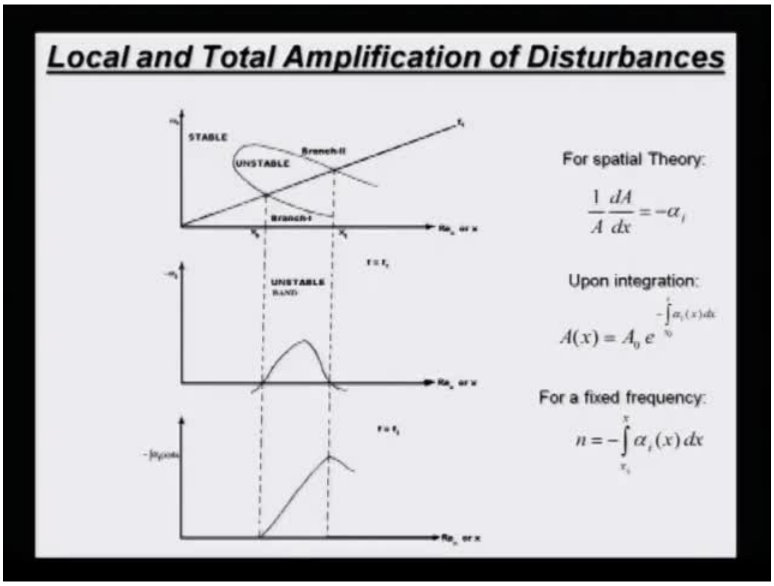

So you create a single monochromatic signal and then trace it, what happens to it. Then you are going to validate whether you have a good theory with you or not say for example, we want to let sat now follow a fixed frequency f 1, and we have already talked about this neutral curve have this thumb shape size and the lower branch is called the branch one, the upper branch is called the branch two; this is the usual terminology. So what happens is if I now plot minus alpha i versus x; so this is minus alpha i versus x then what happens between these two points, I would have got the unstable region, so this is the unstable band and outside these I will have alpha is positive. Now what have we already seen from the spatial theory description that I can calculate this spatio growth rate as minus alpha i which is nothing but one upon A d A by d x. So this is rather easy to do it, d A by A equal to minus alpha i d x, so I could integrate, so l and A equal to integral of alpha i d x, then I can take that. So what I have done is I have written it down that l and A, I have gone from an initial station x naught to some current station, at that initial station x naught let we call that amplitude a naught; at the current station I have the amplitude A of x, so A of x is nothing but a naught e to the power minus of x naught to x A alpha x d x. Why did I put this x within parenthesis of alpha i, again the same thing that we talked about that this is what we are expecting, we are expecting that the flow will latch on to the local condition. That is why we are saying that my theory is based on locally parallel flow approximation, and I have collected those values I have got in those diagrams but then what I am hoping for the flow to do is latch on to this local condition, and then what I am getting is alpha i is a function of Re, because different station has different Re, and then I integrate over it. So this is an exponent minus alpha i d x which I call as n, and now this n that we have obtain is for that frequency so this n itself is a function of that particular frequency, so different frequency will suffered different types of amplification, so this is well of the way that we like to do. So what do you find that we are integrating alpha i d x, and alpha i minus alpha i goes like this, so if I integrate it will go like this. It will continue to grow but the rate is going to change depending on the location but beyond this it will decay. So if I start my datum at this point, in the unstable band this n value is going to increase; it will reach a Plato and again starting falling beyond that unstable band.

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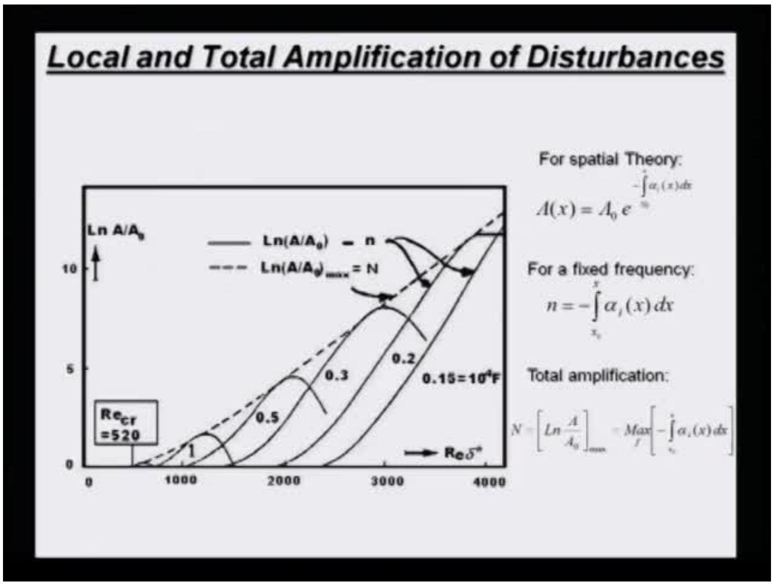

Now, this basically tells us what to expect, well this is what do we expect, things are getting interesting. What happens, again we are plotting now Re on this axis; Re are the stream wise distance, and on this axis what I have plotted, I have plotted L n of A by A naught. So that is essentially that integral minus alpha i d x or n, so this is that n value; lower case n value, then for different frequency I do get this different curves.

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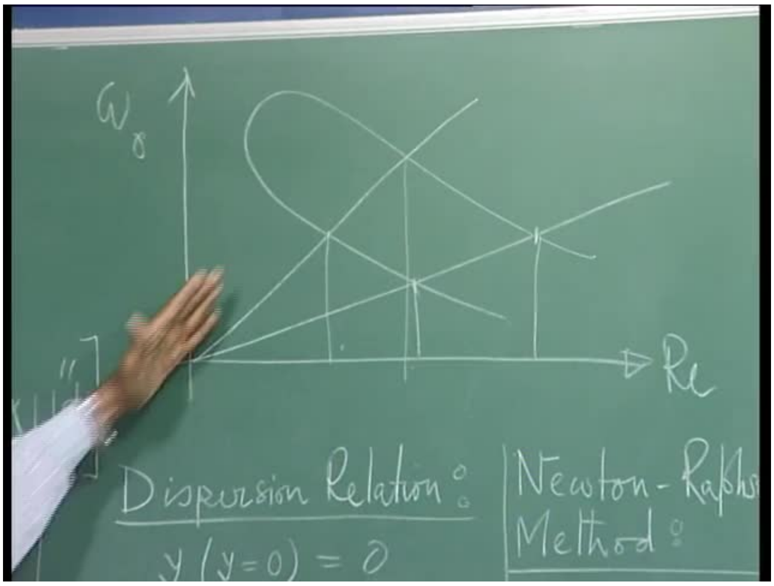

Do you all agree with the general feature of this, yes because you can see that if I drawn the neutral curve; this is the way the neutral curve is. Now if I look at a low frequency then what happens, it enters the unstable region later and exits also later as compared to let us say another higher frequency which should be like this. So that will enter here and exit here whereas, this is entering here and exits there and that is precisely what you are seeing here, the lower frequencies that we are plotting here starts off late, they grow and they exit also late to a larger band whereas, a slightly higher frequency you would see starting off earlier, exiting also earlier.

And what we notice also because the range basically, we are taking a horizontal projection of this, so the extent is going, come down and that is what you also see that as you frequency is increasing the extent of unstable band is decreasing, so for this it is about from one thousand to here, for this it is about let say one core thirty lakhs one thousand four hundred all the way up to about two billion forty lakhs five hundred of that kind. So this is the story that we going to and research, and what we find that all these begin at that Re critical, we talked about close to five hundred and nineteen or something of here we have written five hundred and twenty.

Now what happens is different frequencies are given by this sort of curve, so I could draw, and envelope are shown by this dotted line and this dotted line basically is in locus of all these different maximum that we have for different frequency, that quantity is what we called as capital N. So this is somewhat like this that as I go along, some disturbances are going to grow and different frequencies will have different growth rate as given by this. So we talk about a fixed frequency growth, and we can talk about a total amplification as if that as we are going from one station to another this amplitude is going to translate into the next frequency and so and so forth, then your disturbance has grown along in dotted line, see these all very approximate engineering ideas and what we are talking about this is our L n of A by A naught. So if we have L n of a by a naught as given by that dotted curve which we have called as capital N, so essentially what we are talking about then so that means what, so basically n kind of signifies the exponent which the maxima will tell us.

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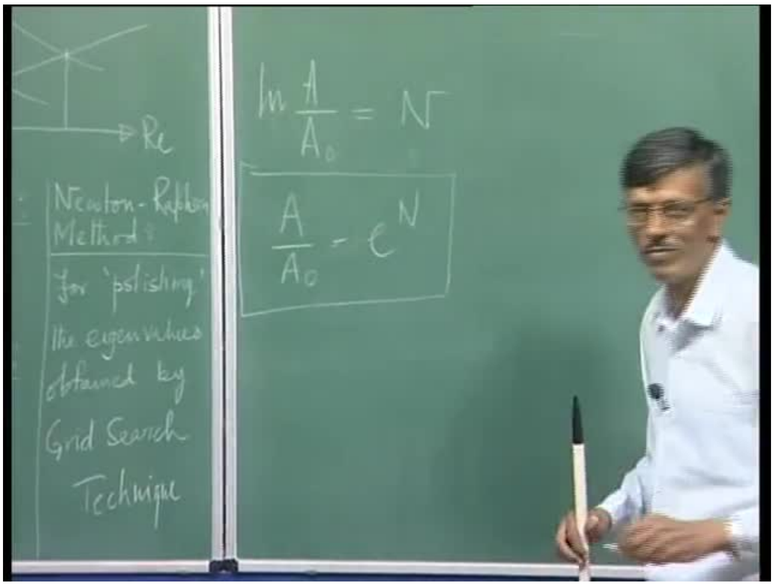

Now, let us say if you think what people do then what is unknown in stability theory, we do not know what is A naught, we cannot say we are talk about relative amplification that is what we are talking about, and that determines on this n curve. If for different frequency we have different lower case n, and the global envelope tells you about the capital N. Now basically, then that is the story of disturbance growing inside. What was a change by this specific experiment of schuhbauer and skramstad was basically identification of that Re critical and then tracking a fixed frequency disturbance, the extent of growth range and so and so forth, those are the success stories of the stability theory. Even today this theory is very extensively used in analysis and design of the aircraft wing. What you do is basically, you try to obtain this n curve, here the n curve that we have obtained is for a zero pressure gradient boundary layer. So, if I am trying to design a wing, think of what we need to do at each and every step say these are all kinds of local adjustment of the flow, and let me also tell you little bit in advance that a pure hydrodynamics scenario like the way we are discussing in this course, things do work out. We just simply found out very recently that if you add e transfer, yes linear theory is stopped working, so do not get carried away because you would be coming across various claims where people would say oh! We use a very sophisticated theory called e to the power n method, and this is your e to the power n method. This is not theory, theory it was a sense that you have idealized the actual flow in terms of a locally parallel flow and you have obtained all of that.

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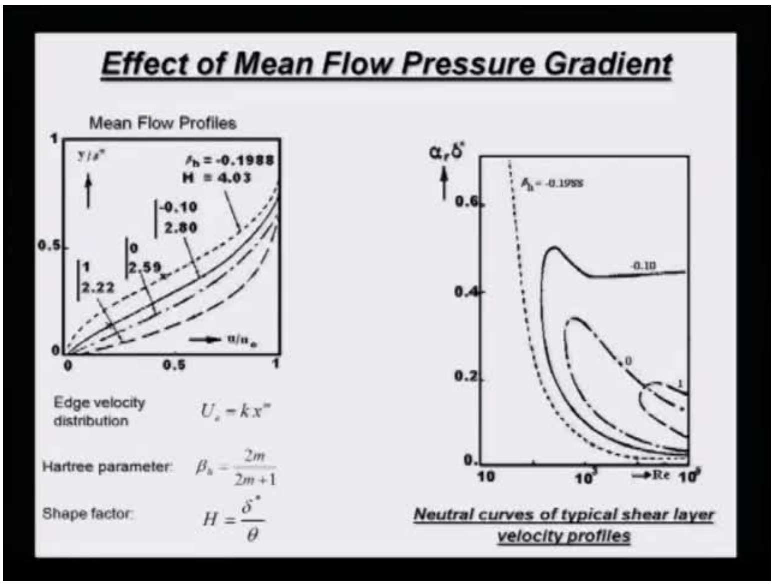

And now in an actual geometry what will happen is you are going to get all kinds of pressure gradients, if you think of so the aircraft wing design I would have at different station different pressure gradient, it is not going to be a flat plate. Of course you calibrate and game confidence from standard geometry and standard result that is the Blasius profile you are happy that you have done it and now if you want to go and use it for actual practical scenario then what you are going to get, you are going to get a variable pressure gradient flow. And what we are then talking about, we are talking about different mean flow profile as we go along over the wing at different stations we going to have different mean flow profile, and here are some files plotted for a benefit.

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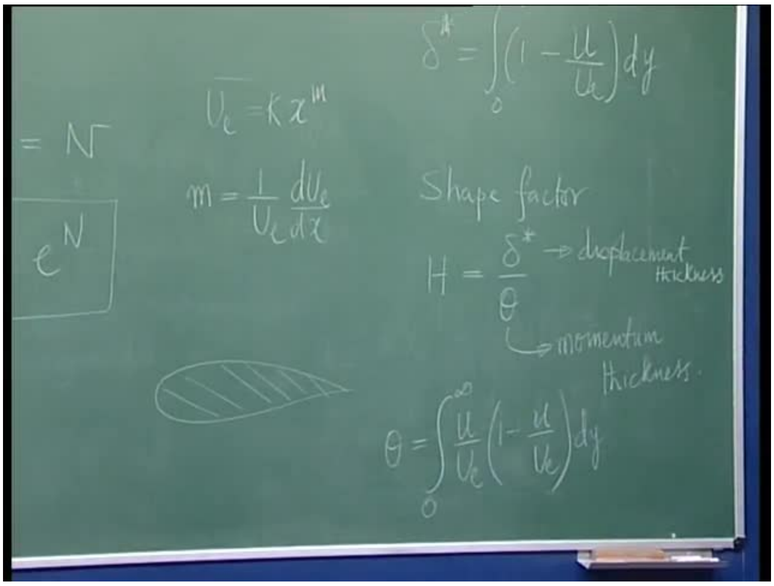

Now this one here indicated by this zero and two point five nine, this actually is your zero pressure gradient boundary layer. How this thing is defined, it is defined by how the edge velocity varies with x, that edge velocity variation is given here by a similarity parameter which is called as(( )) scan parameter m. What is this m, m as we have written you will see it is nothing but one over U e d U e. So what we are talking about is say we are talking about flow pastor aerofoil like this, Now what is happening, as we go long we are going to get a different kind of boundary layer, so I can get U e as a function of x just outside the shear layer. From that U e versus x i can convert the values of m, so do understand that m itself is going to vary along. Now there is another quantity that is of great significance in boundary layer theory called the shape factor, shape factor is called H, and which is nothing but delta star by theta, we have already identified it as displacement thickness; this is what we call as the momentum thickness.

Well the very nomenclature I will suggest what this two quantities are, both are some length scale; we are talking about thickness, one talks about displacement. What does displacement mean, it is as if a streamline has been displace by that amount due to the viscous action, because what happens ideally we would see outside the shear level will have the inviscid stream line but as we come closer to the surface it has to adjust itself so that you satisfy the no slip condition, so this is equivalent to as if the whole flow has been lifted up, displaced that is what is given there. So for an incompressible flow what is the definition of delta square; one minus U by U e d y and this we can do it, so this is what it is. Ideally it should have been equal to one but the viscous effect has brought it down to U by U e, so you integrate over the whole which to get delta star. And theta is defined like this; you are going to tell me what does this those of you who have done a force on boundary layer theory, I hope all of you have done, if you have not then I may like to spend a day or two later, but at this point in time let us talk about what this is. Do we have any idea, any one are you familiar with momentum thickness, have you done it before, you forgotten, you have. So this basically tells you about the momentum deficit; see ideally what was the momentum carried through with the fluid, If we did not have any viscous action it should have been U is half row U e square, but then now what you have locally, you do not have U e but you have U. So what happens is that it is being carried through with the mass given as U row U, ideally it should have been row U e so this first factor gives you about what is the actual real mass flow.

Now the second factor tells you loss in this momentum because of viscous action, ideally it should have been one but it has come out to this, so this is a kind of a loss, this is the actual mass flow times loss, so that will tell about the momentum deficit. So basically, theta tells us about a drag and this component of drag comes about because of the no slip conditions, so this drag would be related to the skin friction drag. So this is what it is, theta gives you an estimate of what kind of momentum that we have loosed due to the shear action, and the shape factor tells you about the coefficient of these two leg scale; delta star by theta. And what we have done here is, now the mystery of those two parameters will come, the first zero that we identified as the Blasius profile, the top zero represents m; m is equal to zero. If I have a zero pressure gradient flow then what happens d U e d x is zero, it does not change; zero pressure gradient, we can see that from Bernoulli’s equation if you are looking at the outside flow. So m equal to zero correspond to Blasius profile, and corresponding H happens to be this number; two point fifty nine, so that is this line; the second last line from the bottom.

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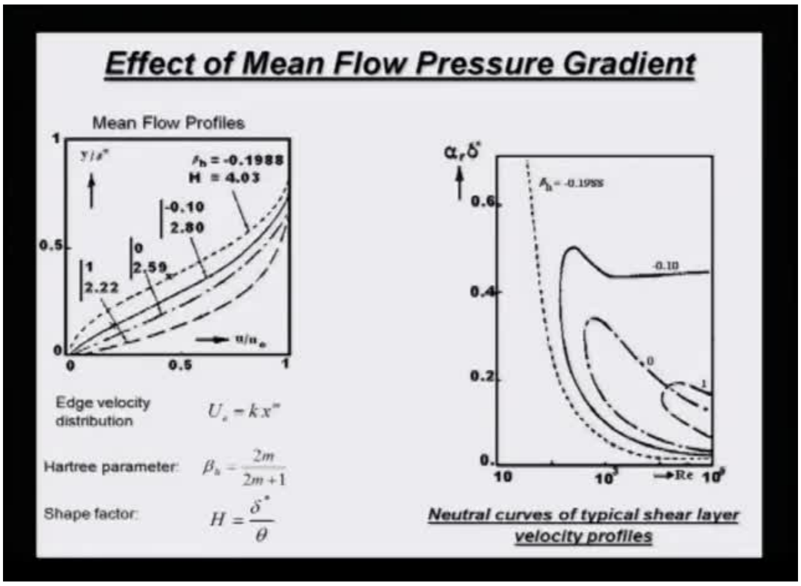

Now what I could do is, I could create a flow of different value of m; what is m negative signify, m negative signify d U e d x, negative means U e is. So what would that mean, that is an adverse pressure gradient flow, so this case then corresponds to a value of m equal to minus point one, and corresponding H stand out to be equal to two point eight. So should H increase or decrease, why it was pressure gradient, you look at the velocity profile that has been drawn, if U was equal to U e then I would have had the profile straight away here one and that it would have gone, so compare to zero this curve is displaced upwards, so it has more mass defect. And then what happens corresponding momentum defect lags so what happens is the numerator increases, denominator does not keep pace with that so overall H actually increases, so that is what you are seeing that m has become negative, so you have an adverse pressure gradient and H has increased. The other scenario would be the case where m will be positive and then the corresponding value of H will be much lower.

Let us first talk about the adverse pressure gradient, little more you can see that because of the adverse pressure gradient the velocity profile becomes deficient closer to the wall where you come to a stage where this curve actually has a slope of equal to d u d y equal to zero. That corresponds to your separation condition; the skin friction locally becomes zero. That is what is the value given here for the top corner; corresponding m is about minus point one nine eight eight and H becomes four point zero three. So what happens is if you keep on having this adverse pressure gradient about (( )) becomes thicker as the flow has a tendency to separate, and that is the limiting value. Whereas, this is the other case; this case m equal to one, we will talk about in the next class. We will discuss it somewhat greater details in the next time.