Instability and Transition of Fluid Flows

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Lecture No. # 10

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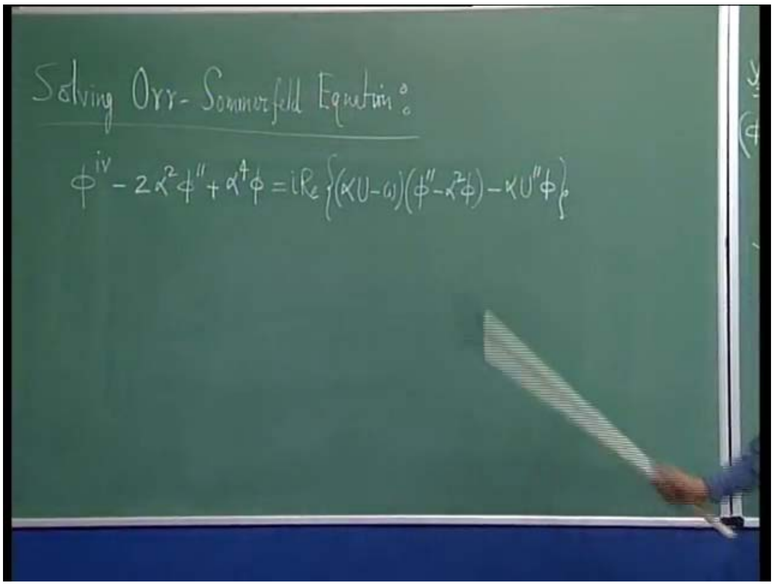

So, let us begin again. Trying to find out method, which will solve Orr-Sommerfeld equation. We have identified its stiffness, in the last few lectures. We also identified a method called the compound matrix method which involves finding out those modes which satisfy a set of boundary conditions. You see the fourth order ODE, we have 2 boundary conditions at the wall 2, in the far stream. So, you can get some kind of analytical structure of the solution in the far stream, because you know their U double prime is 0, this capital U is 1. So, it becomes a constant coefficient ODE and we found out, there are 2 viscous modes and 2 inviscid modes and in each class 1 mode grows with y another decays with y.

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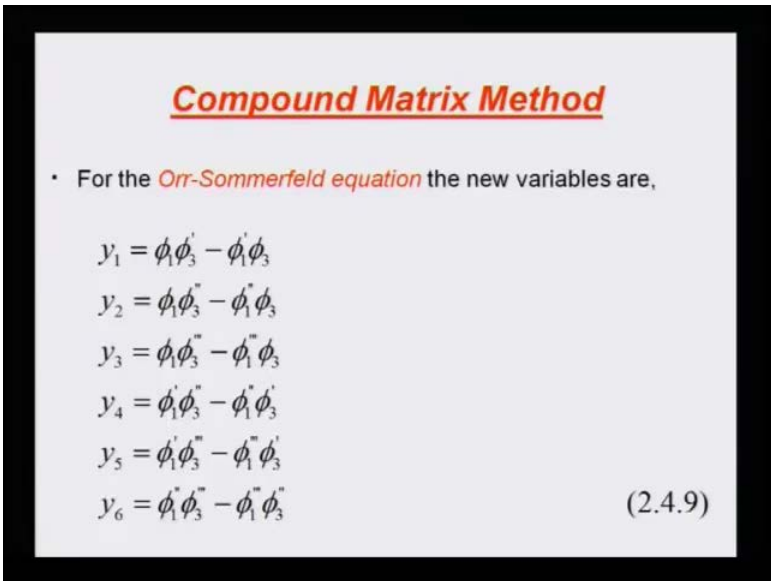

So, if we are solving a problem, where disturbances created inside the shear layer, we would like the disturbance to decay with y and those of the ones that, we have identified as phi 1 and phi 3. And we noted that there is a disparity of scale of variation of these 2 fundamental solutions. phi 3 actually, grows much more rapidly as you descend inside the shear layer, as compared to phi 1. So, this disparity causes the stiffness problem. This was what we concluded and we came with the conclusion, that we will use a method which can do away with this problems. For example, if I define this fundamental solution as phi 1 and phi 3, in the last class we talked about that the knowledge of the solution is equivalent to having this solution term.

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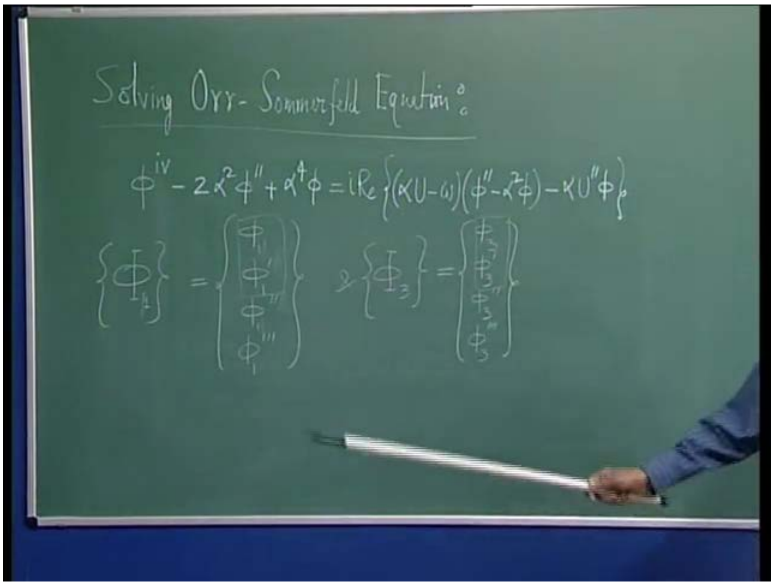

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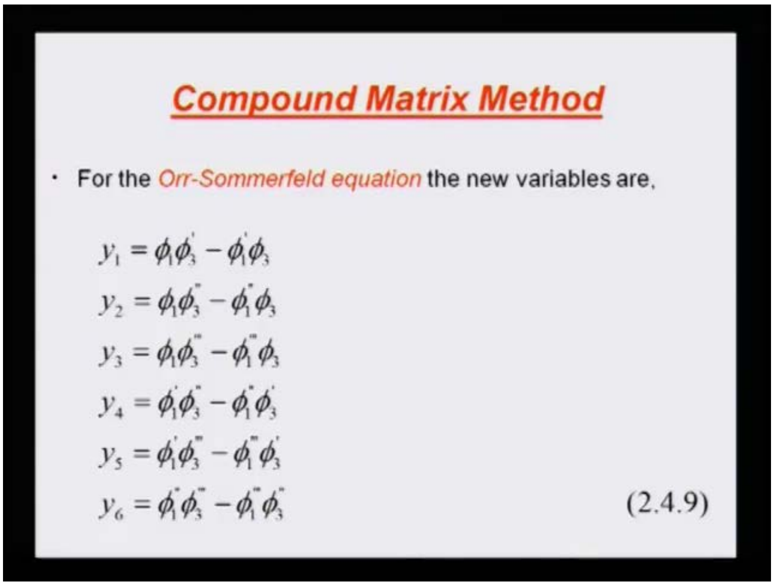

Say for example, if I talk about phi 1 solution vector, that actually corresponds to. So, let us call this as capital phi 1, one of the mode. So, it will have the phi 1 and phi 1 double prime, phi 1 triple prime and of course, phi prime, phi 1 prime, phi 1 double prime and phi 1 triple prime. So, this is one of the mode and other mode is, if I call that as capital phi and write it like this, then we are going to get phi 3, phi 3 prime. So, basically what we have done in defining this compound matrix variable we have taken it, 2 by 2 sub matrices formed by capital phi 1 and capital phi 3. For example, y1 consist of these 2 elements, from here and these 2 elements from here. And that is what, you get phi 1 phi 3 prime minus phi 1 prime phi 3.

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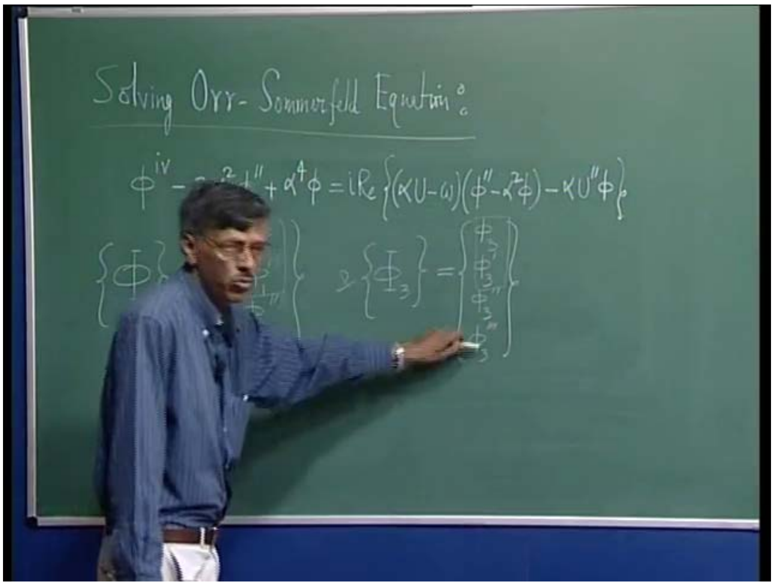

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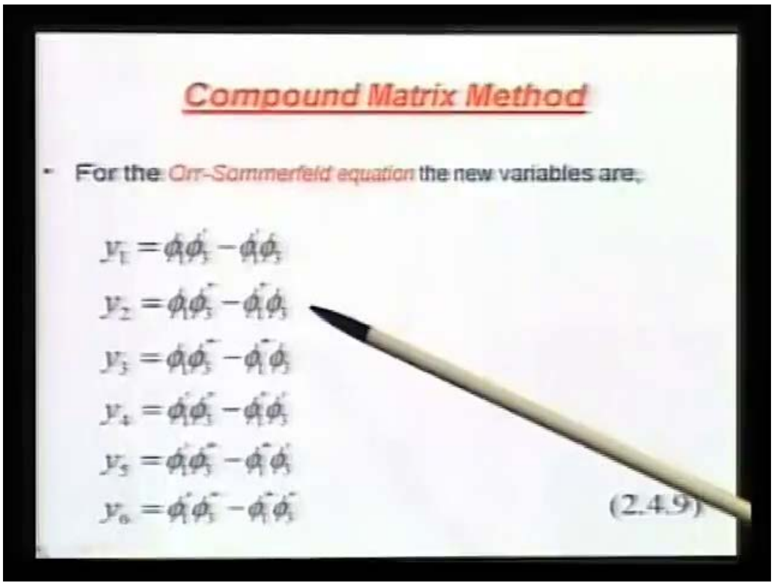

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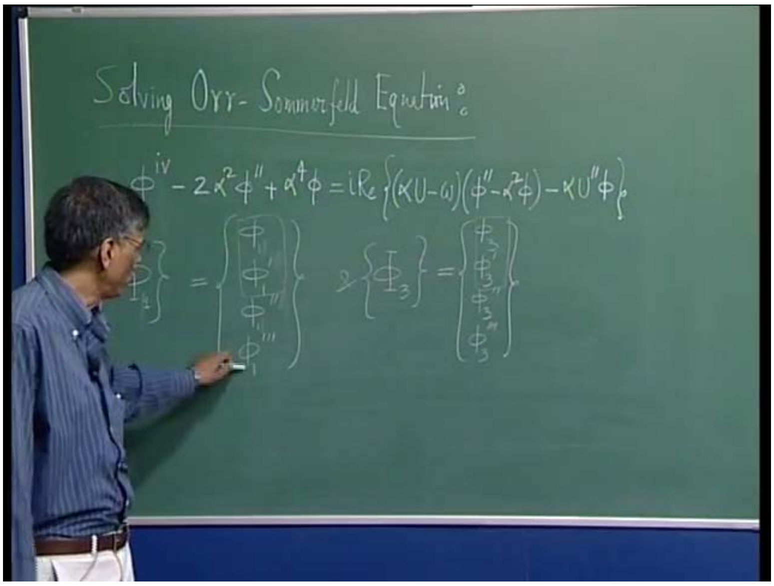

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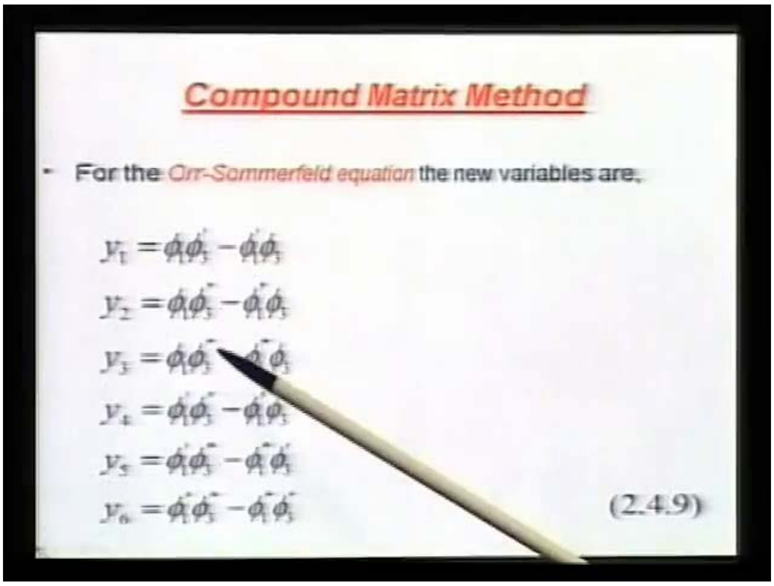

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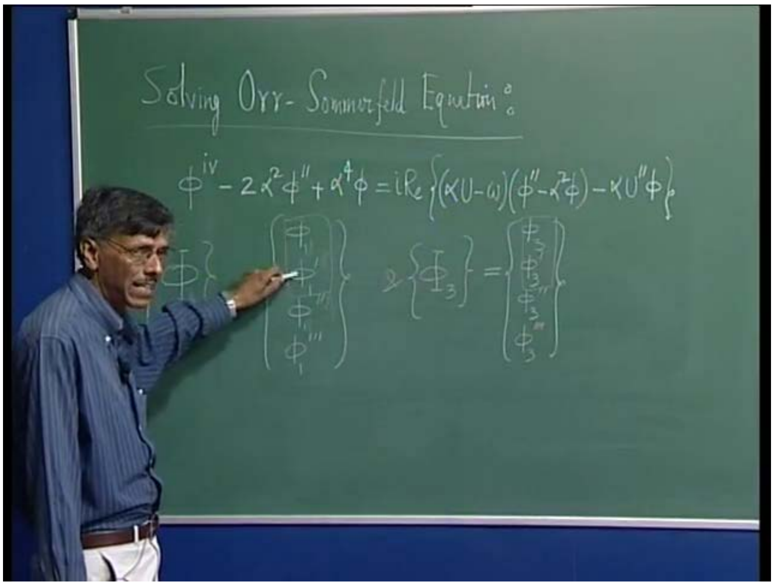

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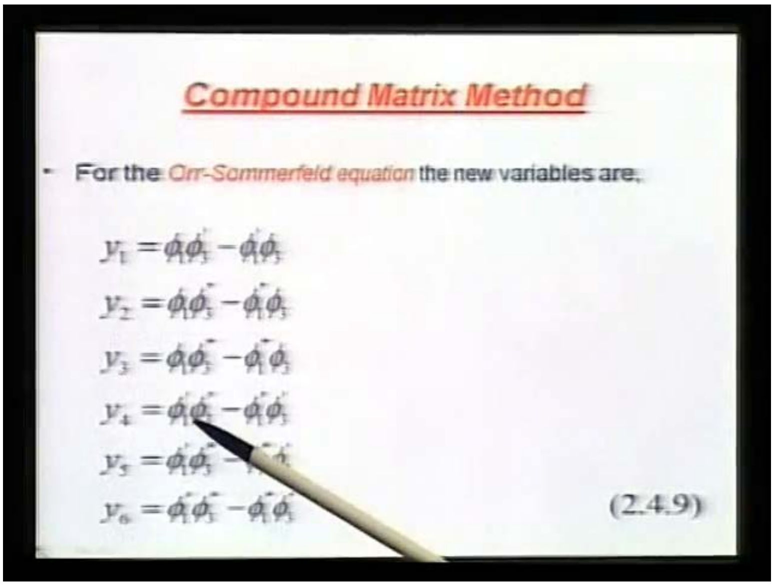

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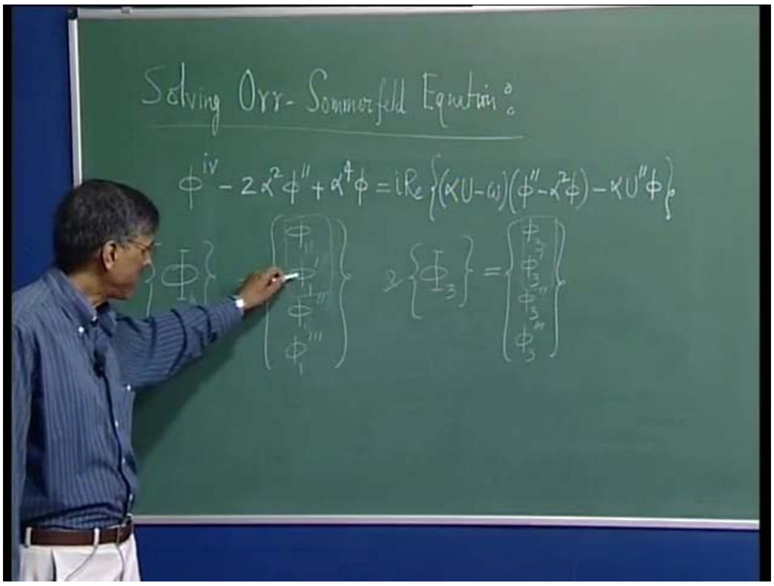

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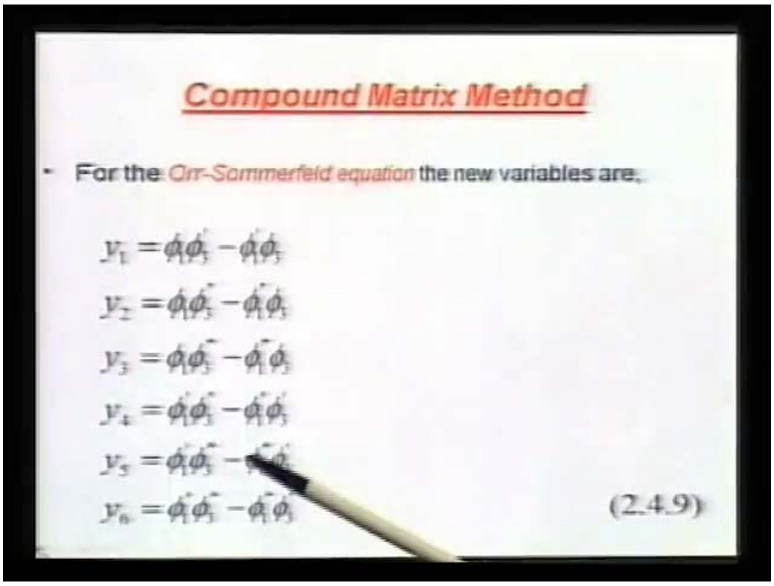

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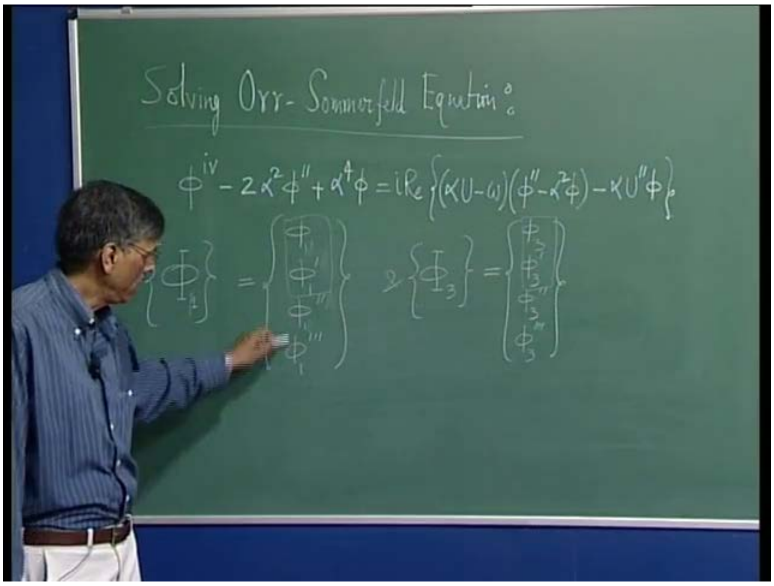

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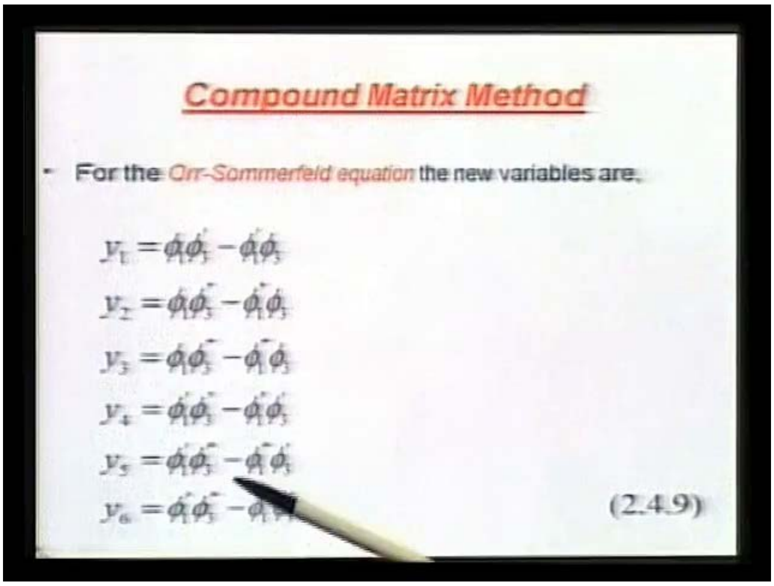

Suppose, I take the first and third, from this 2 vector column vector, I get y2. If I take the first and fourth, from both of them, I get y3. Then, of course, the other possibilities, I take the second and third, that will give me y4, second and fourth give me y5 and third and fourth will give me y6. These are all possibilities that you can conceive of, because it comes from that you have a 4 fundamental solutions, you have picking 2 at a time. So, it is a 4 C 2 number of variables. That is what you are seeing. Now, basically, when we go from the phi variables to this y variables, we need to derive those equations. A set of governing equations for this compound matrix variables. And we have indicated, how to go ahead. By utilizing the definition of the variable itself, for example, if I differentiate y1 with respect to y, that is y1 prime would be nothing but phi 1 prime phi 3 prime plus phi 1 phi 3 double prime minus phi 1 prime phi 3 prime minus phi 1 double prime phi 3. You can see these 2 cancels and this is nothing but the definition of y2 itself.

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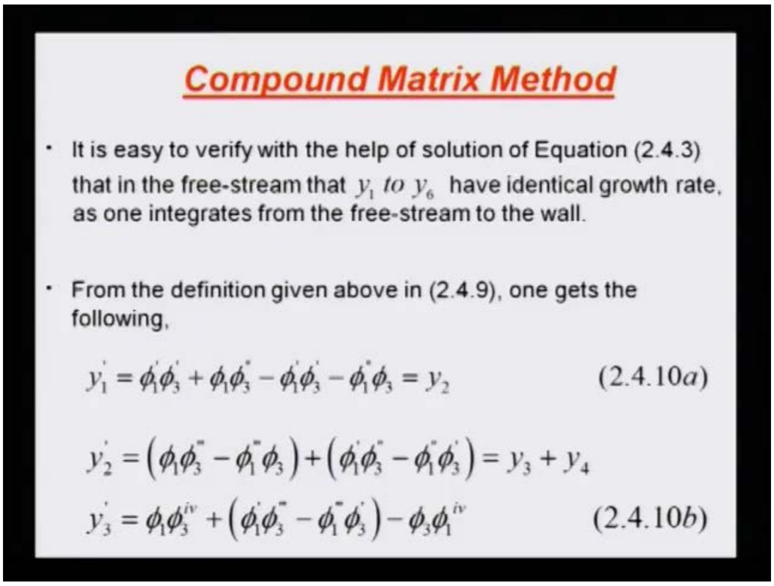

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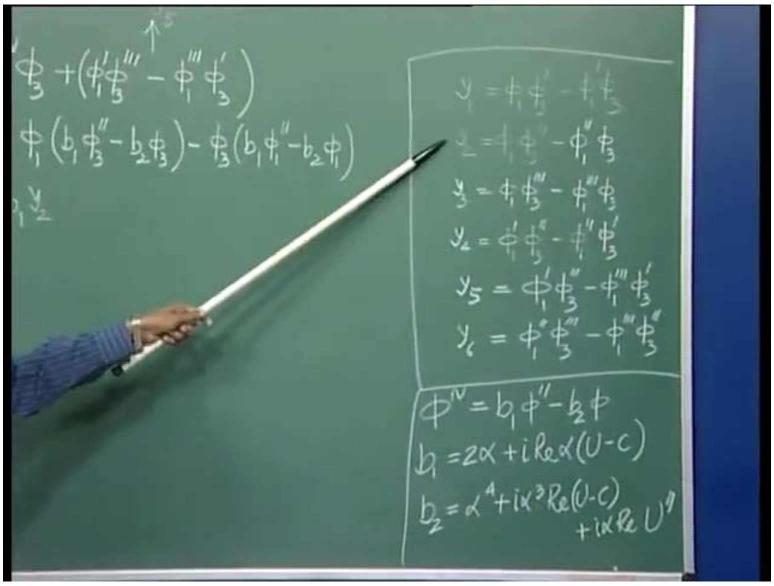

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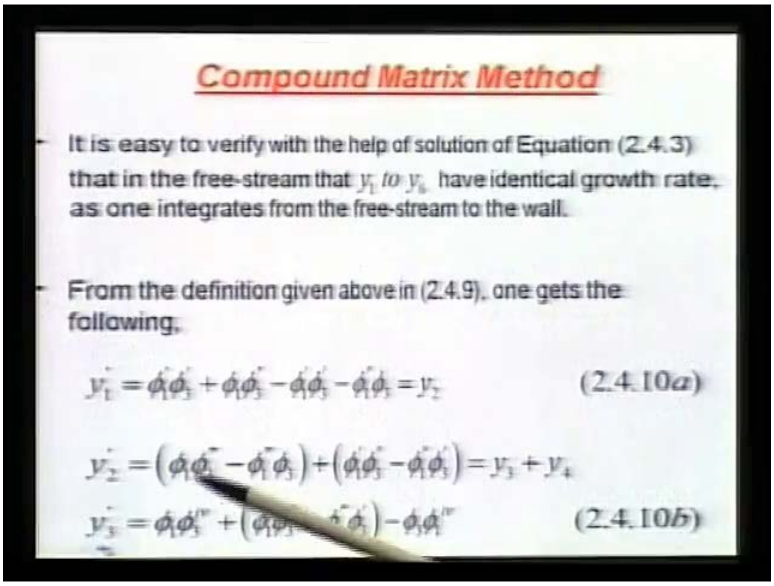

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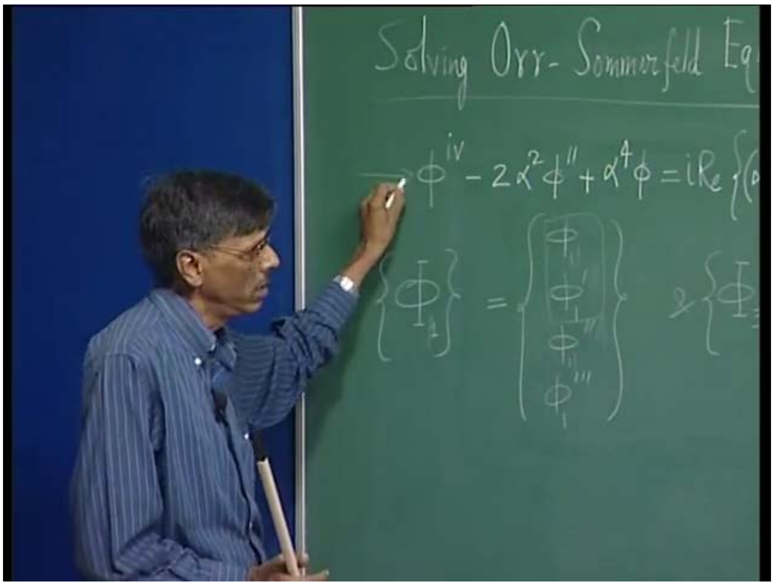

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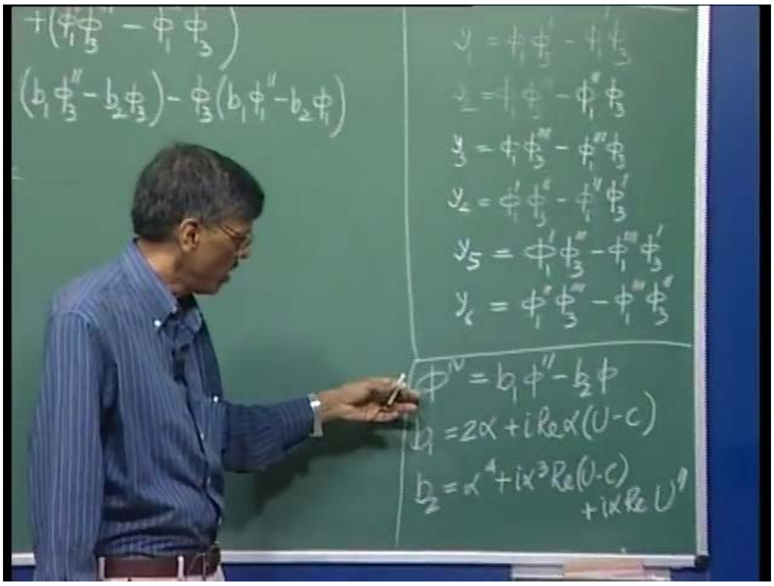

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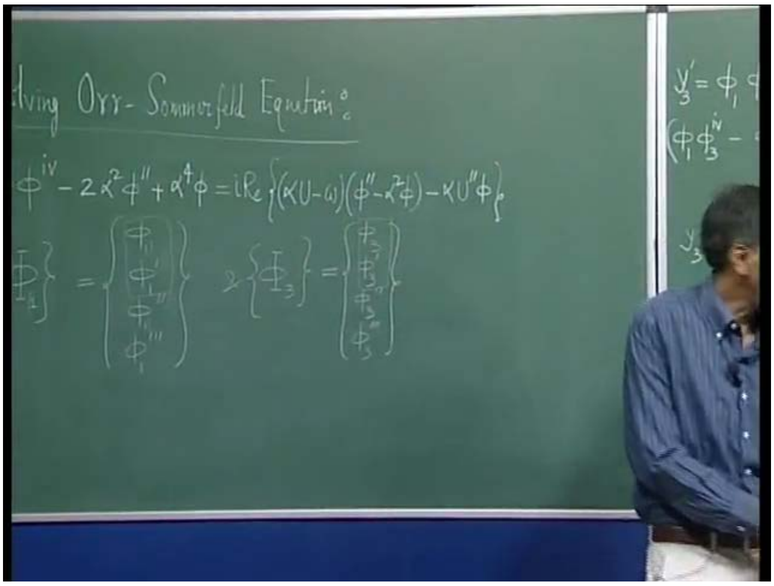

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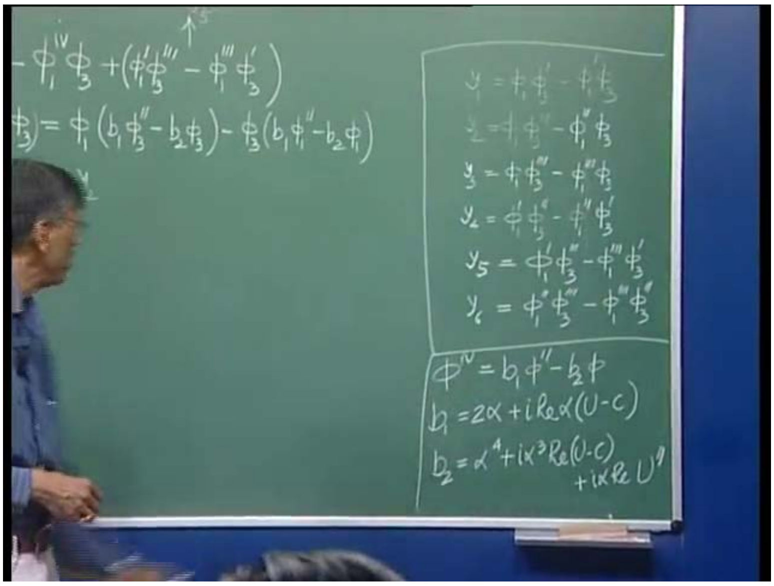

So, one of the equation immediately obtained was y1 prime equal to y2. You can go through the same exercise, using the definition of y2, as I have written it down here. You can differentiate it once and you will get a set of terms, that would involve phi 1 phi 3 triple prime minus phi 1 triple prime phi 3. So, that itself is your y3 and there is the other term, that is phi 1 prime phi 3 double prime minus phi 1 double prime phi 3 prime. That is in a sense again y4. That is what, we have obtained. Now, there is a little bit of extra work that you need to do for the other equations. For example, y3 prime would involve now phi 3 4 prime and phi 1 4 prime. And we know, that individually these modes themselves satisfy this equation and that means what, I can write it in this form that phi 4 prime should be equal to some coefficient b1 multiplying phi double prime minus b2 phi, where b1 and b2 are given. You can take a look at that, you can club those terms together and this what you get phi 4 is b1 phi double prime minus b2 phi prime.

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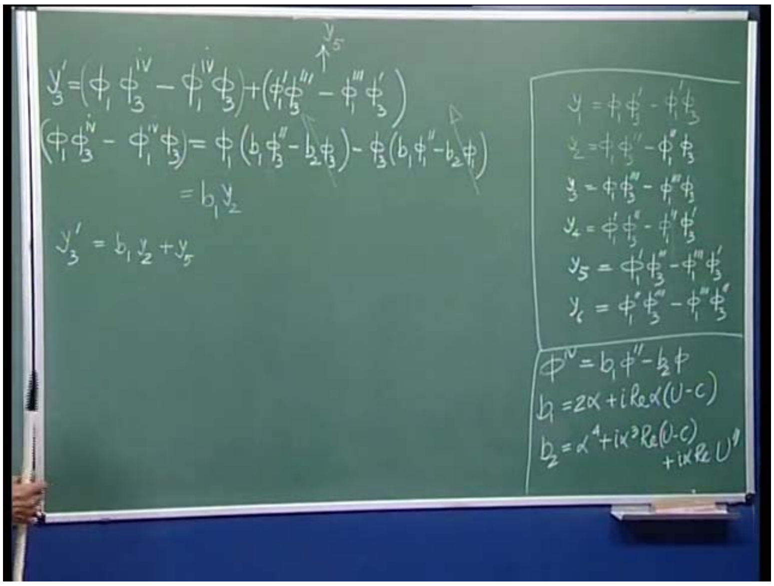

And this y3 prime gives you this set of term. So, this is one set of term that we have and this other set of term. This term is easily identifiable as y5. You can see, phi 1 prime phi 3 triple prime minus phi 1 triple prime phi 3 prime. That is what we get. This other set of terms that we have. Well, for phi 3 4 prime, I just write it in terms of phi 3 and phi 3 double prime, same thing here, phi 1 4 prime, you write it like this. And we see that this term actually cancels with this term. So, what we are ending up with, is b1 times y2. So, that means what, y3 prime is equal to b1 y2 plus y5. So, that is easily understood.

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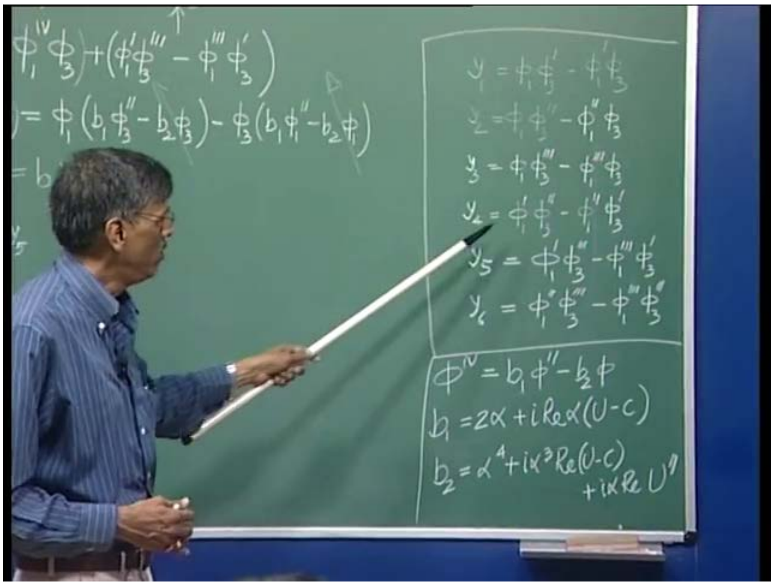

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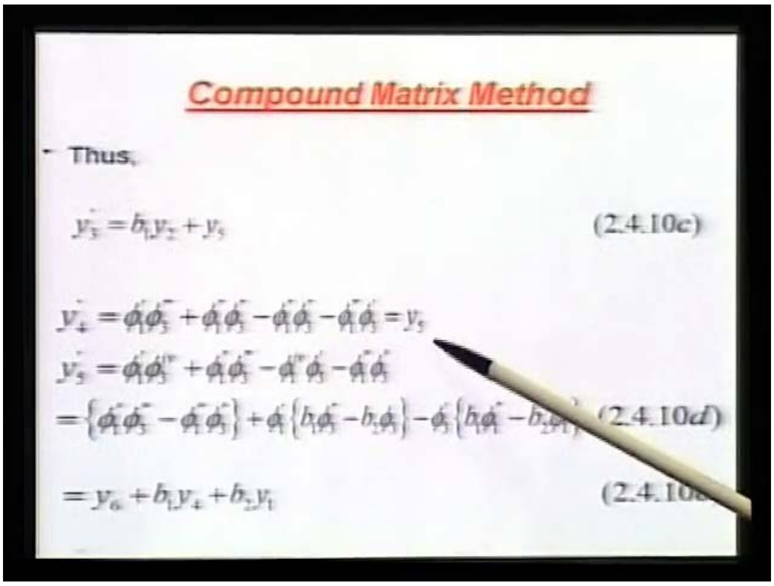

Now, we can go ahead and keep doing same thing. And for y4 prime, y4 prime as we have defined here, as phi 1 prime phi 3 double prime minus phi 1 double prime phi 3 prime. See, if I take X derivative, it would not require the Orr-Sommerfeld equation. You can directly differentiate and that is what you get. That is what is written here. phi 1 prime and phi 3 triple prime and phi 1 double prime phi 3 double prime minus phi 1 double prime phi 3 double prime minus phi 1 triple prime time’s phi 3 prime and these 2 again cancels. You can easily see. And this and this constitute y5. That we have seen. So, y4 prime is equal to y5.

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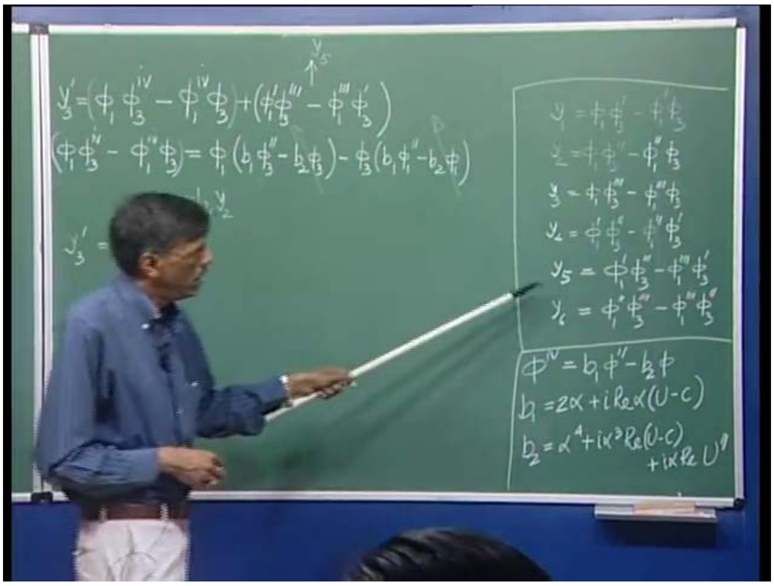

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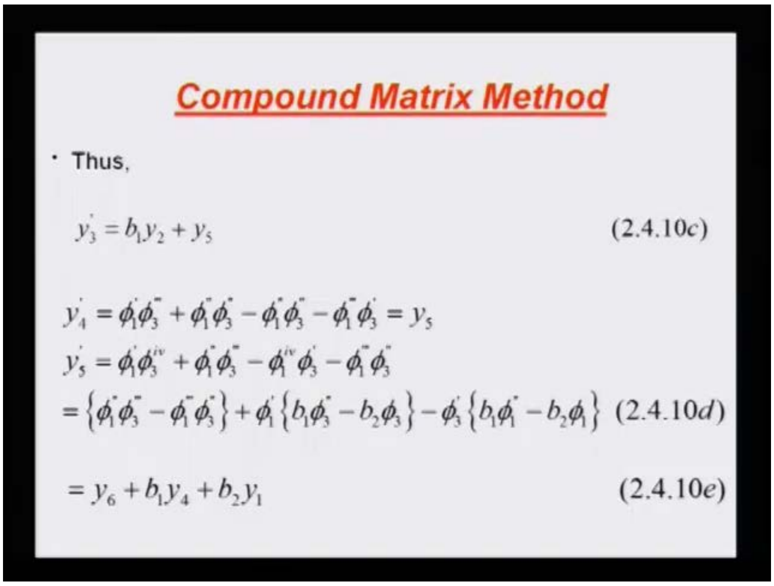

Now, if you look at the definition of y5, it involve the third derivative. So, if you differentiate it, of course, you have to again invoke the Orr-Sommerfeld equation, that we have written here. And that is what is done here. So, y5 prime would be given in terms of this. This first and third term has that fourth derivative. That you use, in terms of that b1 and b2 coefficients. And this one and this one constitutes, so, this is double prime time's triple prime combination, that gives you y6. So, this is your y6. And here you going to have here, look at this. Here b1 into phi 1 prime phi 3 double prime and here, we have b1 into phi 3 prime phi 1 double prime. So, what is that? That is your y4.

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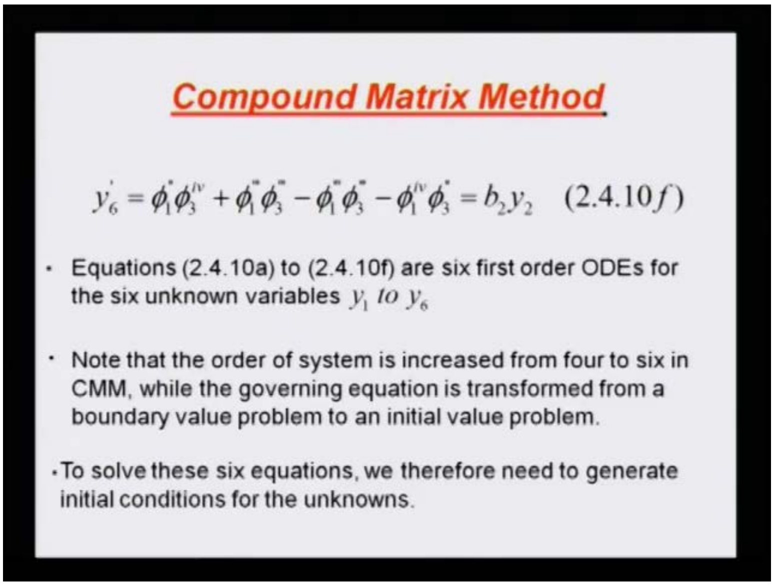

So, basically then, we get a term. This gives us y6. This set of terms give us b1 times y4 and this b2 term is very easily identifiable is nothing but b2 y1. So, I think we have obtained y5 prime and then, we need to just look at y6 prime. And y6 prime, we can through the same exercise and you can reduce it to b2 y2. So, you see this is one of the beautiful aspect of compound matrix method. Our original problem was a boundary value problem. We need it to satisfy a 2 conditions of the wall and 2 in the free stream. By the shear definition of the new variables, we have already satisfied the far field condition. So, now, what we could do is, if we can start from the far field, we can march all the way up to the wall. And then we need to satisfy that condition. So, basically what we have done is inverted a boundary value problem into a kind of an initial value problem, because we have to satisfy one set of condition only.

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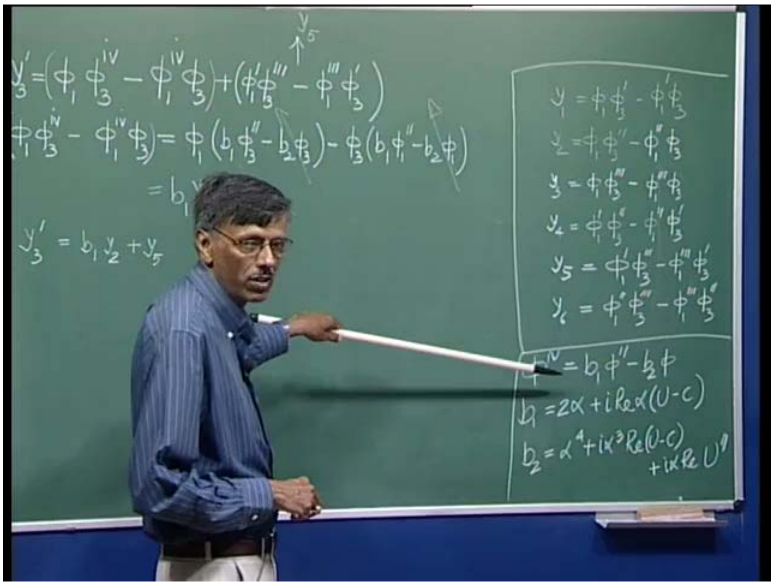

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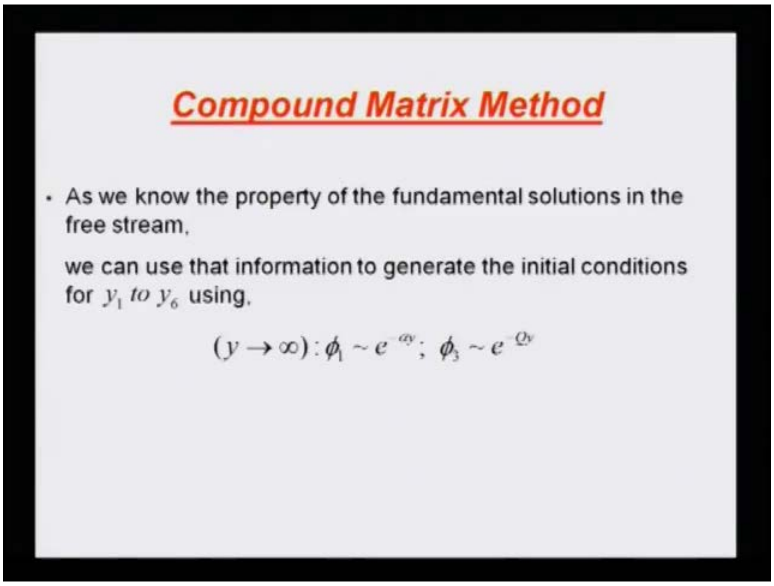

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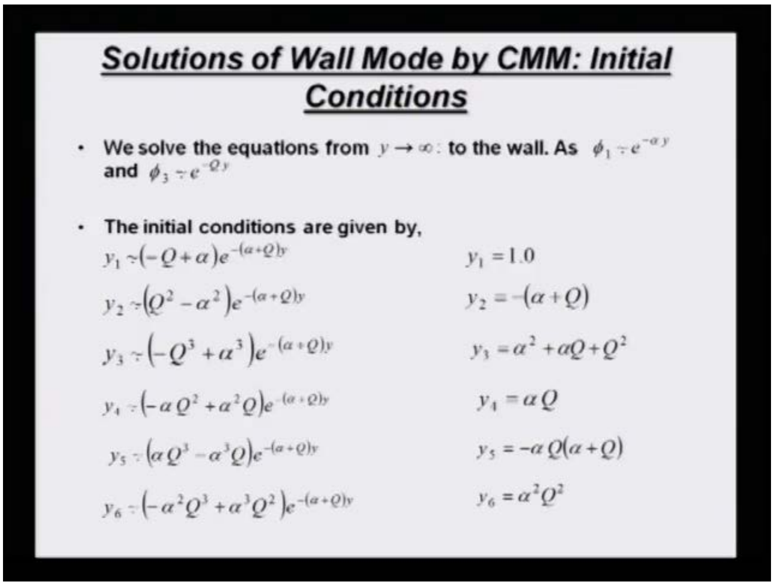

So, this is something what we need to do. And however, we make this observation that in doing so, we have increased order of the system from 4 to 6, because now we have sixth coupled first order equation. Now, as I told you that, we could start marching from the free stream and we could march up to the wall. But the question is what do you do as the initial condition for these unknowns? The definitions are here. y1 to y6 are known here. We also know, how phi 1 and phi 3 varies in the free stream. phi 1 varies e to the power minus alpha y and phi 3 varies as e to the power minus Qy. Q has that expression Q square is equal to alpha square plus iRe into 1 minus C. So, we can actually, get to use this analytical structure of the solution for y equal to infinity and obtain them, initial conditions. Let me reemphasize to you again, that we are looking at problems, where the disturbances are created inside the shear length. So, that kind of disturbances give rise to, what we called as a wall mode, because the disturbances are predominantly near the wall and they decay away from the wall. So, that is what we said. So, if we now do that, we talk about the wall mode. phi 1 is e to the power minus alpha y and phi 3 is goes as e to the power minus Qy.

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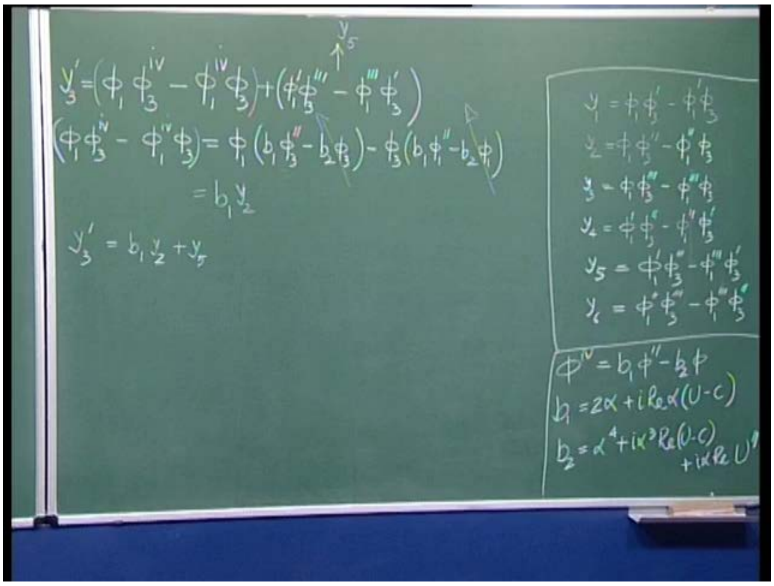

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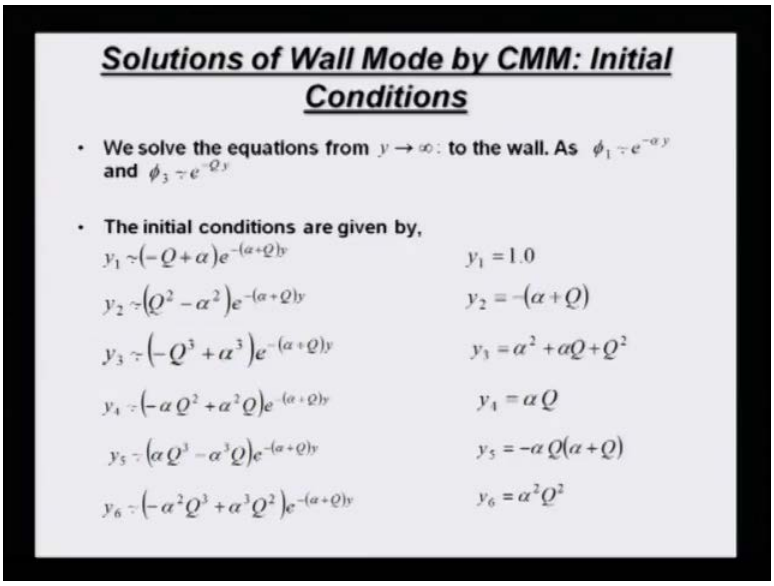

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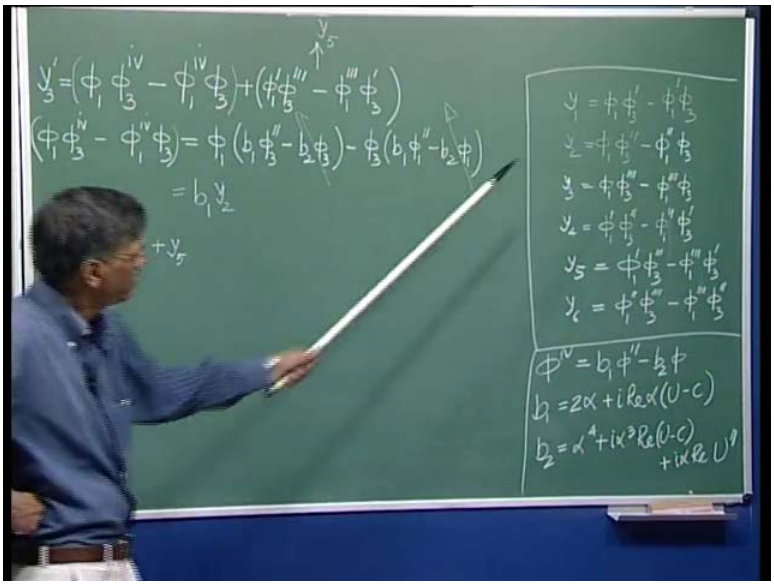

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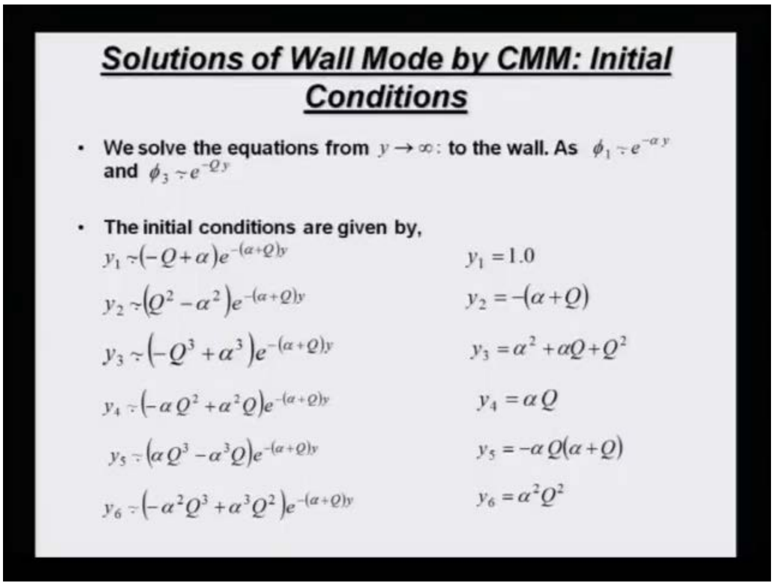

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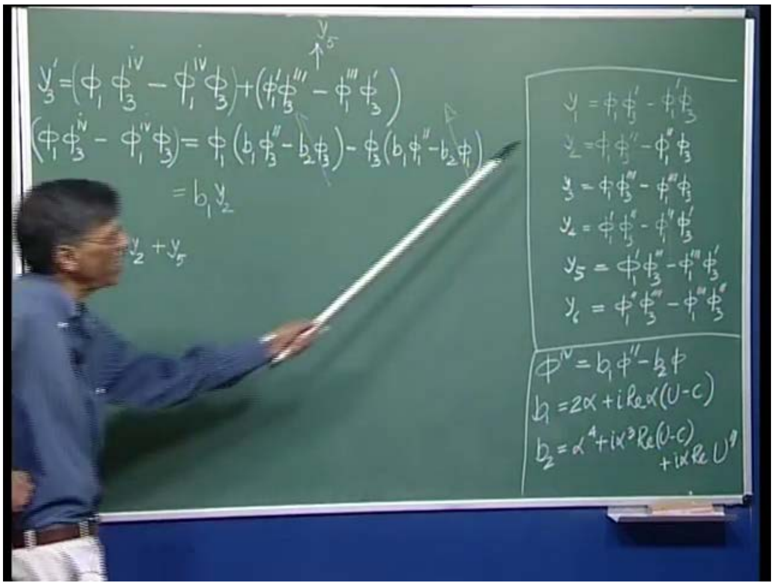

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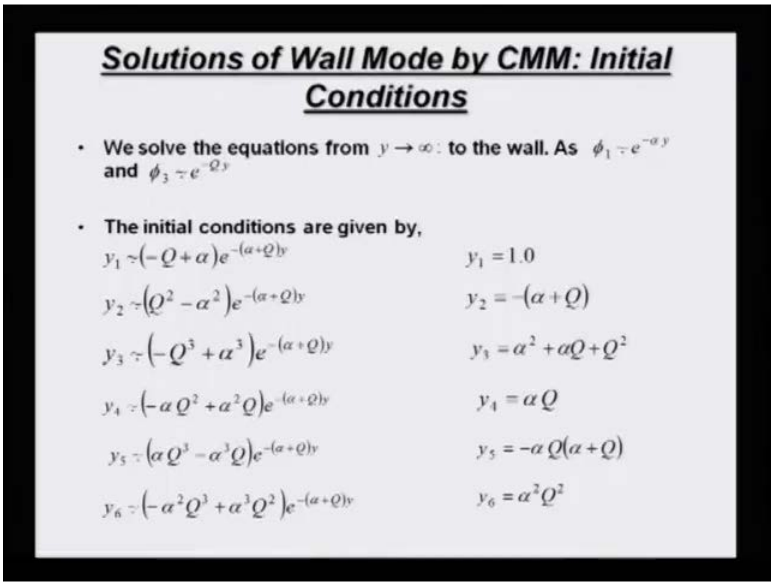

So, I can use those definitions, then y1 works out to minus Q plus alpha e to the power minus alpha plus Q into y. That is easy. You can do that phi 3 prime will be nothing but Q times e to the power minus Qy. And this is of course, already e to the power of minus alpha y. So, those 2 product will give me this exponential quantity. And you notice that, this gives me that exponential quantity. This also gives me the same thing. That is what, we have done. We have pulled it out and this is what, we get. Same thing, go ahead and plug those expressions for phi 1 and phi 3 and put them in here. y2 will be nothing but Q square minus alpha square times the same exponential factor. y3 is this, y4 is this, y5 y6. Now, take a look at this. What we promised, we deliver now. We said that we are going to define a new set of variables which will have identical growth or decay rate. And that is exactly what you are saying here. The exponential part, they are identical.

What matters is basically is, this coefficients that remains affront. And you know, when you are solving a stability problem, what you know is the quality of the solution, you cannot exactly quantify. So, that is why I did not put it an equal to sign here. I said y1 goes like this y2 goes like this. So, exact multiplicative constitute is not known, because we are solving a Eigenvalue problem. We are not solving a receptivity problem. In the receptivity problem, we would have prescribed the quantum of excitation of the wall and then, we could have worked along that line. However, you also notice that, I could factor out alpha minus Q from everywhere. One of this quantity times this, because this is the variation. So, I could normalize everything with respect to y1. Then y1 would be equal to 1. y2 will be what? It will be just simply minus of alpha plus Q. So, this is it. So, you can go ahead and get this. Now, this is what will constitute your initial condition.

You have now a value of y1 to y6, you could start far outside the shear layer and start marching downwards. So, it is a fairly easy task. And what has happened you know, you do not have to anymore worry about those stiffness. You have actually factored out, this stiffness by this clever design of the variables. So, what you have is a Sixth first order coupled ordinary differential equation with very neatly given initial condition. You can use any method of solution. You can use any method.

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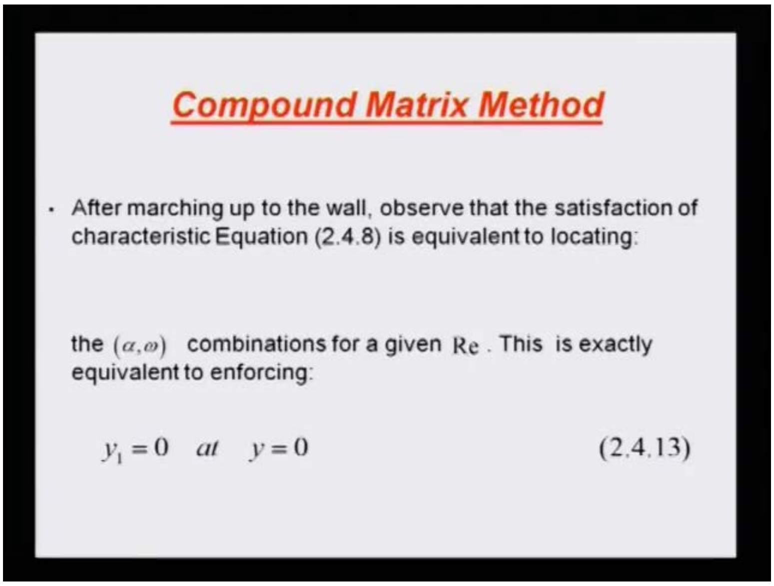

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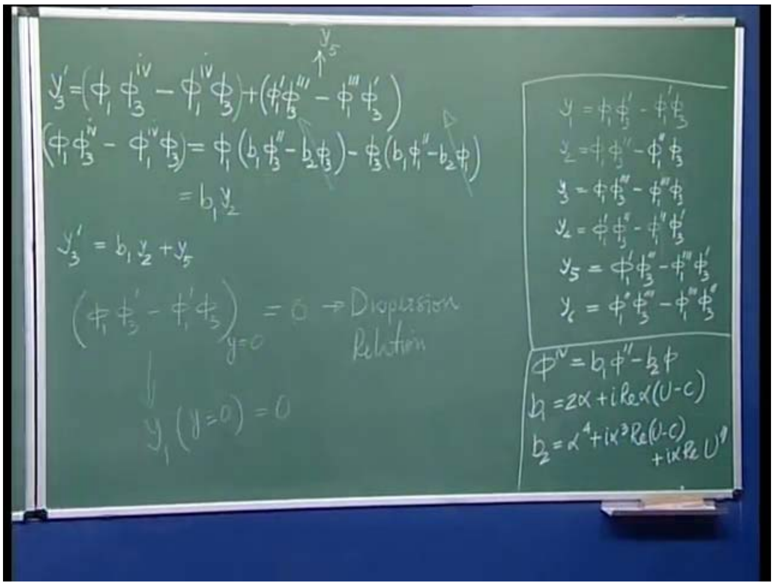

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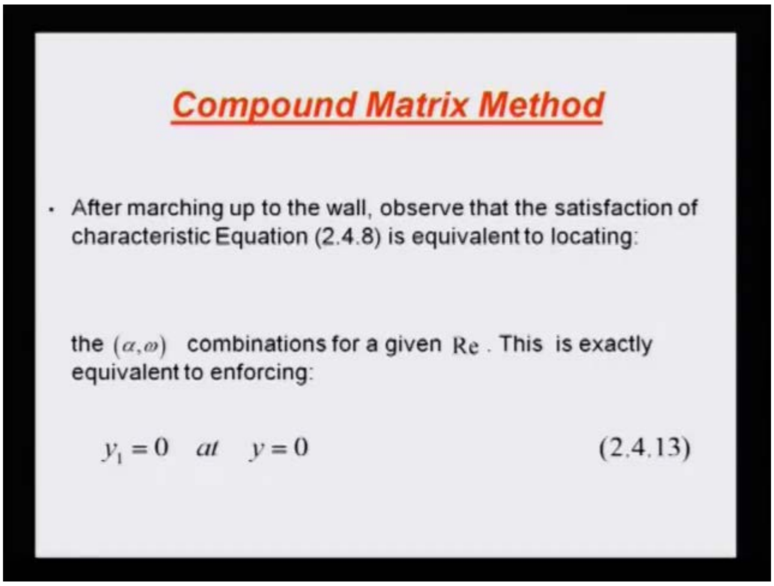

So, what people have been waiting for long time, nearly almost all 60, 70 years, that was fulfilled by this compound matrix method. In one go analytically, we have removed this stiffness of the problem by clever design of the method. This is the reason, that we started looking at this. And what we need to do is, that we start off with those initial condition in the free stream and we keep marching to the wall and what is it, that we will have to do, at the wall? Recall, at the wall we had the dispersion relation. And the dispersion relation was phi 1 phi 3 prime minus phi 1 prime phi 3. And this was evaluated, at the wall and that has to be equal to 0.

So, this is our dispersion relation, because we noted that not all combinations of omega and alpha will ensure this. Only a selective ones will do that and those selective ones are the Eigenvalues. So, this is essentially your dispersion relation. And what is this? Look at the definition, this is nothing but y1. So, what we have to ensure as a dispersion relation is that y1, at the wall should be equal to 0. That is what we have said. So, satisfaction of the dispersion relation is equivalent to locating alpha omega combination, for a given Reynolds number and this is equivalent to satisfying, that y1 should be equal to 0 at the wall.

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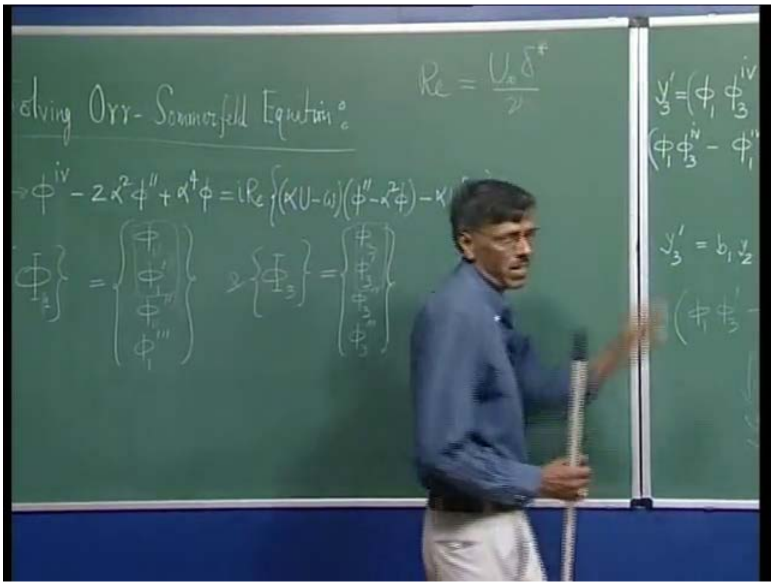

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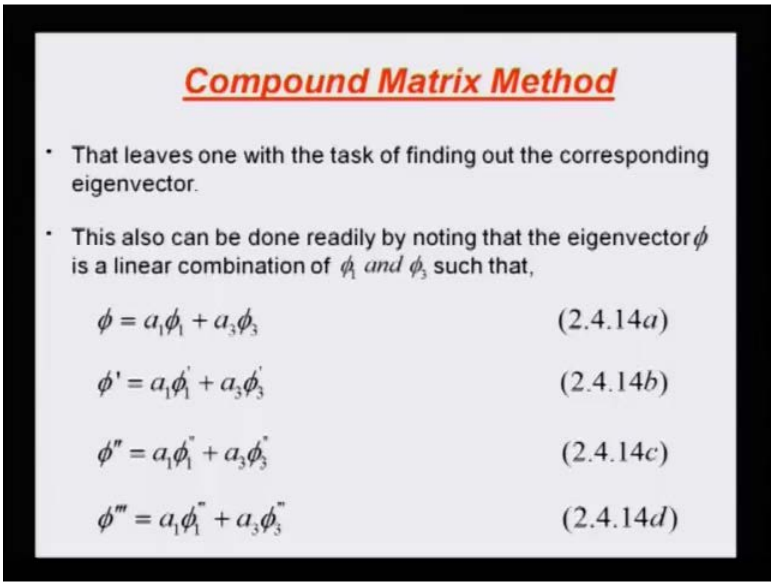

So, that it settles all your problem, I suppose. And that is the way. So, what you are going to do is you fix a Reynolds number, let us say, you want to find out, what is the instability mode at a given stream wise station. So, the Re that we have defined, what is the Re? Re, we have defined here, is a sum velocity scale which I may like to call it, let us say U infinity than times delta star by nu. What is delta star? Delta star is the displacement thickness. So, if I position myself at a stream wise station. I know, what is the local displacement thickness, I know Reynolds number. Now, the game starts. We want to let us say study the spatial stability problem. So, we also fix omega. That is a real quantity. Now, what we are after. We are after a complex value alpha. That is what, we will have to do. So, we will choose some value of alpha and then, we will solve those 6 set of ODEs, come all the way up to the wall. And if we see y1 is equal to 0 there, then the guessed alpha was correct Eigenvalue. If it is not, then again, we will have to conduct the search.

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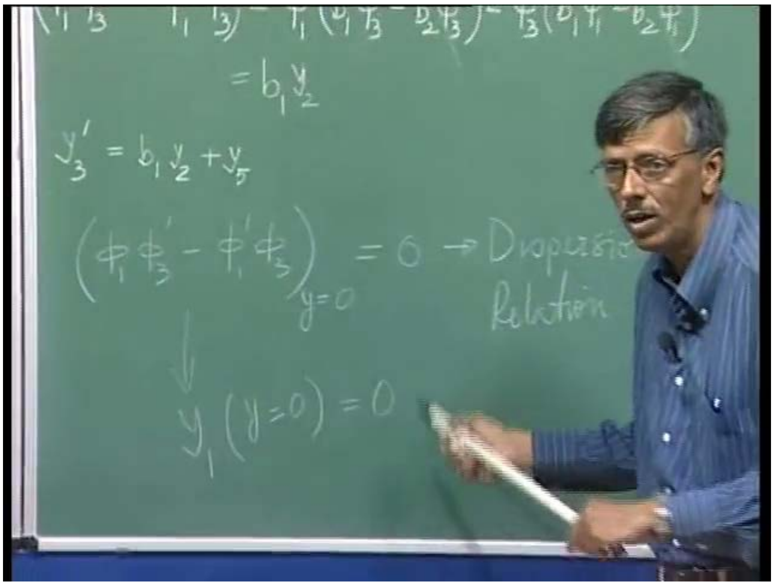

So, if I try to do that, I have an option. There are few ways of doing it and I am talking to you, about a method which is called grid search method. So, let us talk about this particular approach, where we want to find out Eigenvalues. Now, talking about Eigenvalues, you realize that there is not necessarily, only 1 value of alpha that you are looking for. There could be many. There could be for a given Re and omega, you could get many combination values of alpha, real and alpha imaginary that satisfies, this dispersion relation y1 at the wall to be equal to 0.

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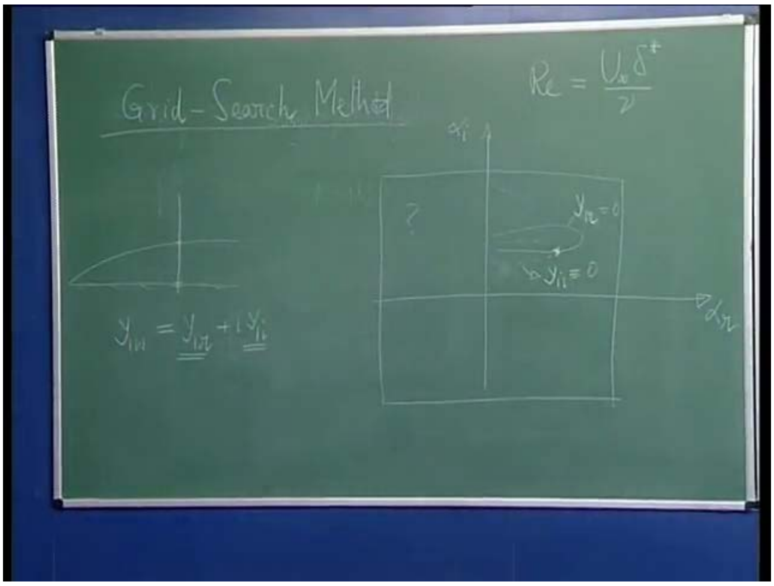

So, what we are talking about is finding all possible values of complex alpha, that will enable us to obtain all the Eigenvalues in one go. So, what you do? This is the problem. Your boundary layer is like this. So, what you do is, you are investigating at this station. So, you will assume that locally, that flow is parallel. That is why, we are talking about U only function of y. So, we will think of, as if there is an equivalent constant thickness boundary layer. So, that is your parallel flow of approximation. Now, what you are doing is, you are starting off with this initial conditions. You march up to the wall and you get y1 at the wall, which is nothing but, I will call, let us say y1 real plus y1 imaginary. Ideally, it should be equal to 0. So, what I could do is I could look up the complex alpha plane, along this I will plot alpha r, along this I will plot alpha i. now, what I want to do is I want to find out the Eigenvalue, let us say, Eigenvalues in this box. So, what I do is, at random I pick up a point here. For this value of alpha, I come from the free stream all the way to wall and I evaluate this. And if it is not a genuine Eigenvalue, this will not be 0. It will always be 0, when your choice is the correct one, corresponding to Eigenvalue.

So, what we can do is, we can catalog that y1 real and y1 imaginary as a function of alpha r alpha i. So, what you could do is in this whole area of interest, you can obtain the value of y1 real and y1 imaginary. So, we are not going into the cumbersome business of trying to find out, exactly where this is 0. So, then what we do is of course, we have the values of y1 real and y1 imaginary here. We can plug the contours of y1 real and y1 imaginary and which contours are we going to be interested in? We are going to be interested in the 0 contours. So, basically what I would do is, perhaps so, let us say I will get a contour. So, this let us say, corresponds to y1 real equal to 0. So, this is let us say y1 real equal to 0. I got a contour like this. There be many such contours, not only 1. I mean, they are it could be filled with all this places. Now, I also can plot this 0 contours for y1 imaginary. Let us say, I get a figure like this. So, this is like your this. Now, you look at this particular point. What is special about that point, that I marked to the nester? It is nothing but the point, where the both the real and imaginary parts are simultaneously 0. That is your Eigenvalue.

So, without worrying too much, I could look at into the whole grid and find out where simultaneously the real and imaginary part of y1 r and y1. So, I can get a collection of Eigenvalues in that box, I have chosen. This method is called the grid search method. So, we can do that. Suppose, I give you a velocity profile. Let us say, talk about the simplest possible case, Blasius boundary layer, I will give you U as a function of y, the capital U and ask you to do this exercise. You can figure it out. Now, you could stop me and quiz me about this part. What is this part? This part corresponds to alpha real equal to negative, but so, far all along, we have avoided talking about it. If alpha real becomes negative, then what happens ? You see, we have this modes. Let me spend a little time in bringing this aspect, because this was not pretty much obvious well into 90s, till we talked about it and we really poked our finger on everyone's eye and say, look, this is the way you have to interpret your results.

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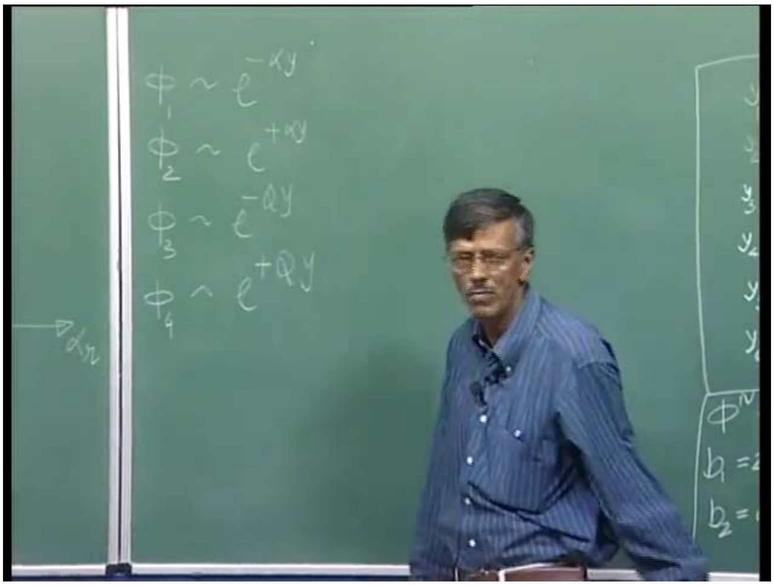

You know, you will see lots of papers. There people say, how there are Eigenvalues here and they do not even want to talk about it. And at the most, one of the paper I saw, they put in some kind of a question mark. I give, it is there, it is not known. So, in that contest we came into the picture and what we said, that look our phi 1 goes as e to the power minus alpha 1, phi 2 goes as e to the power plus alpha y, phi 3 goes as e to the power minus Qy and phi 4 goes as e to the power plus Qy. Now, so far, we have been talking about when the real part of alpha was positive. That made as choose phi 1 for the wall mode. But suppose, the real part of alpha is negative and what I have to do nothing, I just choose phi 2 instead of phi 1.

It is basically that and what will happen for that disturbance. That disturbance, will still decay with height. So, you can still get the wall mode there. Now, what happens is, the same way I could choose between phi 3 and phi 4, corresponding to what I want to do. So, whatever we have done, qualitatively we do not have to do anything different. Only, just be conscious. Aware of the fact, that phi 1 and phi 2 have flipped role. What phi 1 is to do earlier, now phi 2 does that. And what phi 2 is to do earlier, phi 1 does that. You do not have to do very much about that. All the thing, that you need to do is look at your initial condition. Your initial conditions now, need to change, because there we choose, we obtain those initial conditions based on the choice of phi 1 and phi 3.

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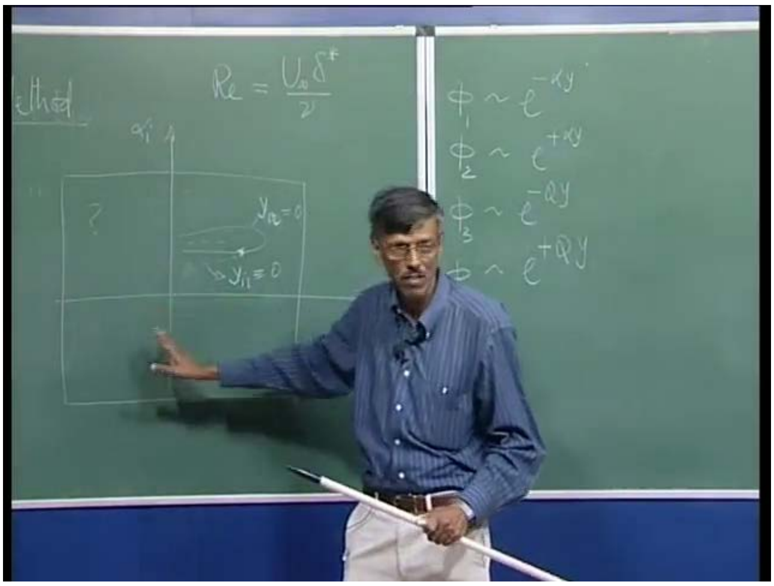

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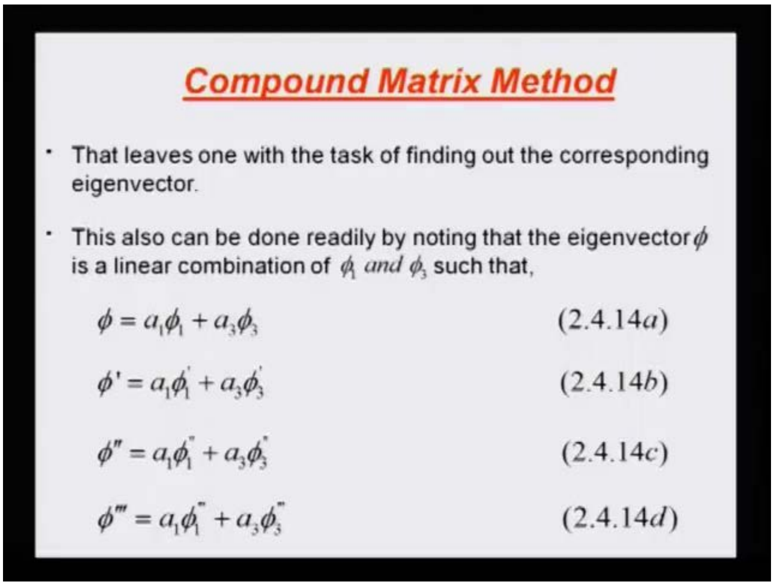

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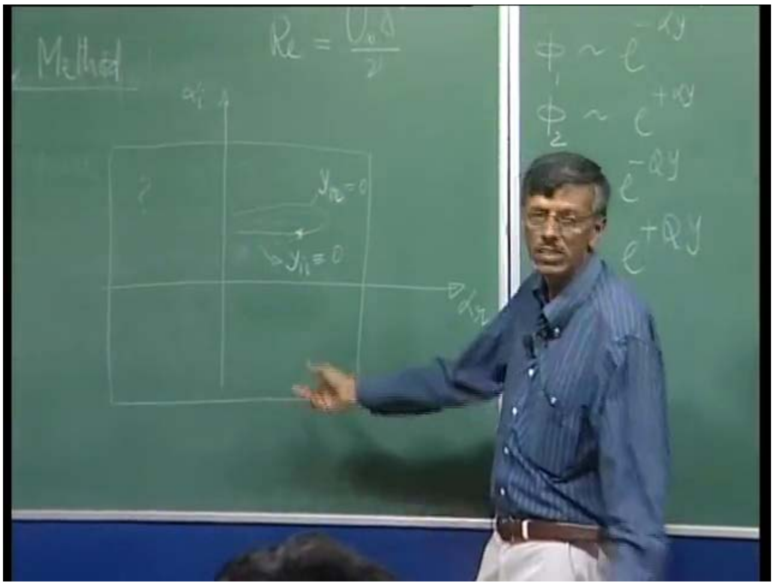

But suppose, I now talk about the left half plane. This is the famous left half plane, which all of us forgot, till mid 90s. So, this left half plane, if I have to look at, I just simply have to rework on the initial condition and then, I can use the same methodology. And I can do the same exercise. So, it is very easy now for you to really do this, grid such method. All you need to do is, two sets of initial conditions. If alpha r is positive you do what we saw in the transparency and if alpha r is negative, we will have to make appropriate modification. And then, we again go through that same exercise come from the free stream to the wall and go ahead and do this exercise. So, I can automate this whole process and find all the values of and this is what, actually we communicated in one of the paper and conference paper. It did not even publish it. I thought, it was quite trivial transform. It should have invoke the curiosity of anyone looking into the picture. And we obtained Eigenvalues on the left half plane. As far as, the phase is concerned, it may omega remains the same and alpha has become negative from positive, what happens to the phase? Phase speed is going to flip sign.

So, in one case if the phase was travelling to the right, in the other case it will be travelling to the left. You can do the same exercise with the group velocity, which will not necessarily be concomitant to it other phase speed. You will have to work it out. We did work it out. And we found that some of this Eigenvalues in the left half plane also travel off stream. What was important or relevant is to find out the importance of those modes, in the sense, the imaginary part. The imaginary part decides, whether it is stable or unstable. And as luck would have it, they are all very stable. So, people who are ignorant, but it did not heard them. So, for 0 pressure gradient boundary layer, it became a pedagogic exercise for us, to at least establish that, if we have the Eigenvalues on the left half plane, we can work them out it. So, happens that they are stable. However, I am not guaranteeing, nobody is that if you change the mean flow this property will remain same.

So, you can have actually, upstream propagating modes. Upstream propagation with respect to the group velocity. Group velocity will determine, whether it is going downstream or upstream. Please do not be convinced by looking at the phase speed. Phase speed is nothing. All of you understand, this property of transverse wave. What does phase mean? The particle going up and down, about the mean position and the relative position of the motion is given by the phase. So, maybe if this point is up, this point is going down. So, this is anti-phase to this. And when you look at this beautiful dance of this particles, it appears as if something is going there. That is what we call as a phase speed. But what is important is, how is the energy transmitted during this transverse oscillation? And this energy transverse speed is given by that true velocity. That is what, we figure out, because that is the speed at which the amplitude travel and amplitude square provides you a measure of energy. So, energy propagation speed is given by the group velocity.

So, it is always proper and correct to evaluate your group velocity at the site for your yourself, whether your disturbance is going upstream or downstream. So, coming back to the third, what I was just telling you, that we cannot approve a guarantee that always the upstream propagating waves are going to be damped. It depends on the health of the mean flow. For a 0 pressure gradient boundary layer that is a Blasius profile, we found that this was damped. But, if we keep having flows with other strain rates, like a pressure gradient, I can change the pressure gradient, thereby I can change the mean flow profile and I could get different types of Eigenvalues spectrum. Collection of all these Eigenvalues together are called the Eigenvalues spectrum. So, the spectrum will change depending on the extra strain rates like the pressure gradient. What else we could do? We could change the mean flow by say heat transfer. We could also do it by mass transfer.

So, anything that you can work on mass, momentum and energy transfer can change your equilibrium flow and you study the stability of that equilibrium flow. One can make actually a safe guess, that if you have severe adverse pressure gradient chances are, you will have a upstream propagating mode. Is not that what separation suggest to you? What is separation? A flow goes like this and there is a pocket, where actually have a recirculation. Means what? It goes in the opposite direction. There is a better work left for some one seriously thinking of knowing a dissertation to relate separation with instability and the corresponding half string propagating modes.

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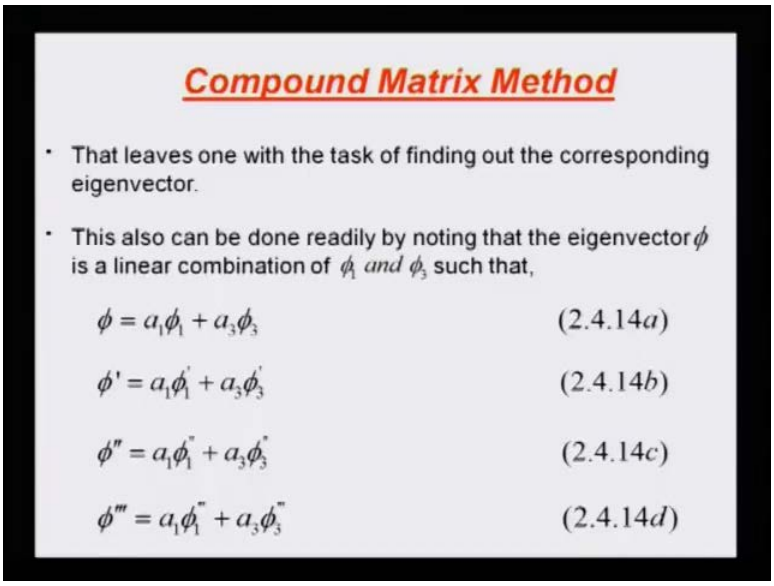

So, it is a very valuable problem that one could look at. So, I think we have talked about the Eigenvalues and their meaning what soever. What is left off? We still have to talk about the Eigen function? How would this Eigen functions look like? Now, what is the Eigen function ? Well, Eigen function is nothing but that amplitude of the v disturbance component velocity, that we wrote in terms of a combination of phi 1 and phi 3. So, if for a problem, I know what this a1 and a3 is and if I can contract to get phi 1 and phi 3, I can talk about, what phi is. That is my Eigen function. Unfortunately, in adopting this compound matrix method, we did not find out phi 1 and phi 3. We found those y1 to y6. So, what do we do? (( )) it is not very difficult for one to imagine, what to do is that having defined phi like this, I could obtain its derivatives, like phi prime phi double prime and phi triple prime.

They all constitute the solution because your Eigen function is governed by the Orr-Sommerfeld equation. So, any derivative less than the highest derivative constitute a solution. So, these are 4 possible candidate solution components. What is unknown here. a1 and a3. So, I could eliminate a1 and a3. Solve for it and plug it in there. And then what happens? I get an equation for phi, in terms of this phi 1 and phi 3 and then, derivatives. phi would not do it, instead I will ask you to do it and submit it to me. This would be your second assignment, to evaluate this 4 possible combinations of solutions. So, basically then, there are many ways of eliminating a1 and a3. 4 are listed here. Is it all inclusive? Well, I will let you pounder about it and decide for yourself. This as all possibilities, or there is something that is left behind.

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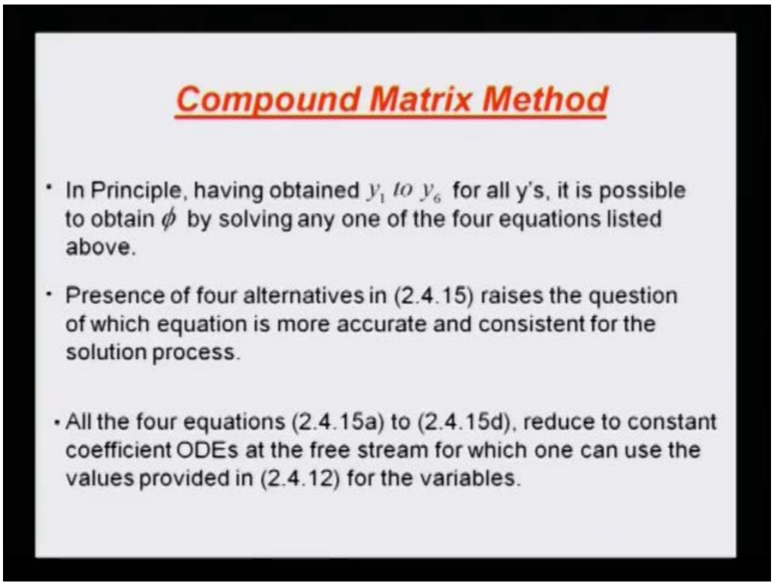

I will give you a hint, this is all there is to it. So you prove it, that it cannot be more than this. You can do that, what you notice is those compound matrix variables that, we have evaluated on our way going from free stream to the wall, they appear as coefficients there. So, what we have? We have this coefficients also stored as a function of y, y1 to y6 as a function of y. So, there are 4 possibilities. I do not know, if I have talked about it here or not, but I will ask you a question. I think, we have just talked about it, in words. Now, I will explain to you what means, that having obtained y1 to y6, in principle we can obtain 5 by solving any one of those 4 equations. Of course, whenever you have too many options, you are in a quandary to find out, which one would be preferred. And preference would be in terms of accuracy, also in terms of consistency. Why do you worry about consistency? I want to you think about it, I will talk about.

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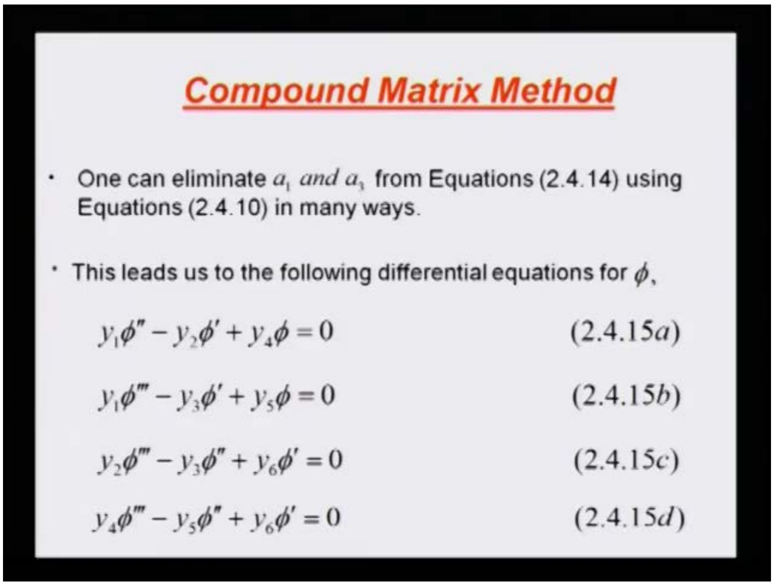

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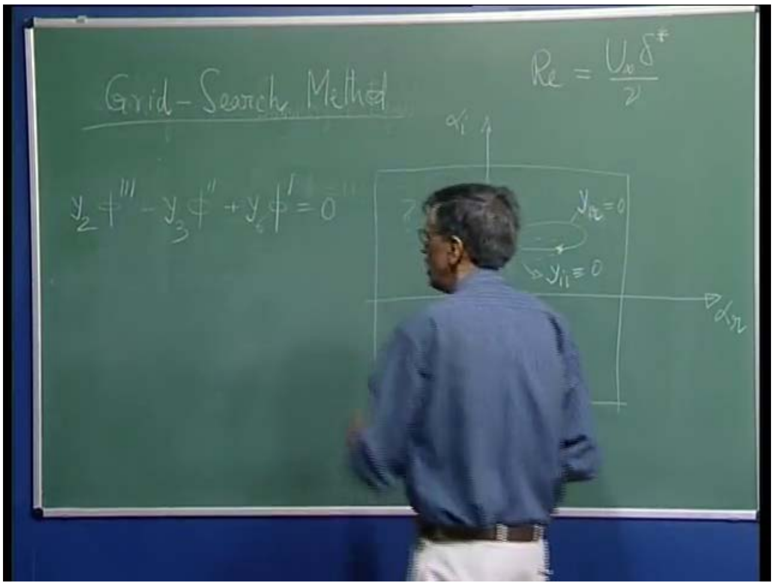

In the following way, if you look at those 4 equations, the coefficients are y1 to y6. So, if I am looking at those 4 equations, in the phase stream, I have an estimate for y1 to y6. That is what, we have done it in the beginning of this lecture, today. We got those initial conditions. So, you can pluck those things. We at least as far as those 4 equations are concerned, we can look at the property of the solution of those 4 equations, because they turn out to be a constant coefficient ordinary differential equation. And if it does, then we can obtain the characteristic exponent of the solutions. I think we have a bit of it here. Say, look at the third equation. Let me just go back and write it down somewhere. So that, we can look at it and discuss about it. The third equation as given here, is y2 phi triple prime minus y3 phi double prime plus y6 phi prime equal to 0.

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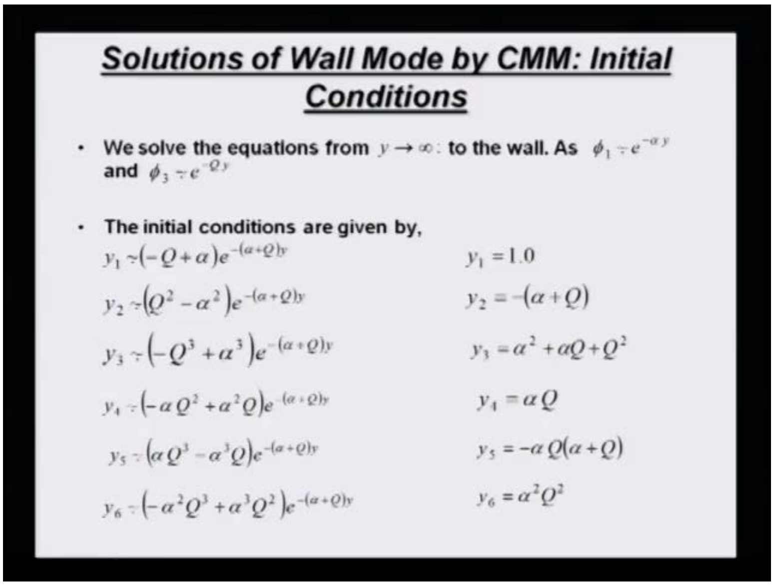

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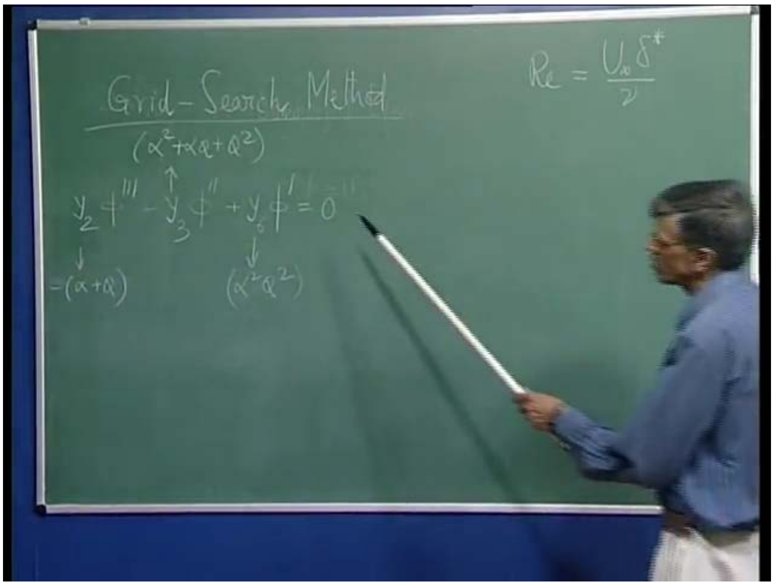

Now, if I look at those initial conditions earlier, what did we have? Let us go back and check them out. There it is. So, what happens is, this is your minus of alpha plus Q and y3 is alpha square plus alpha Q plus Q square. And y6 is alpha square Q square. So, what do we say about this equation? The coefficients are identified for you. So, these are those 3 coefficients and you will have 3 roots. But, wait a minute. What did we start doing with the compound matrix method? We only talked about 2 modes. The once for the wall mounts which decay with y and this is a third order equations. So, it will have 3 modes. But the good news is, you see, this starts off with the first derivative. So, one of the mode is neutral mode e to the power 0.

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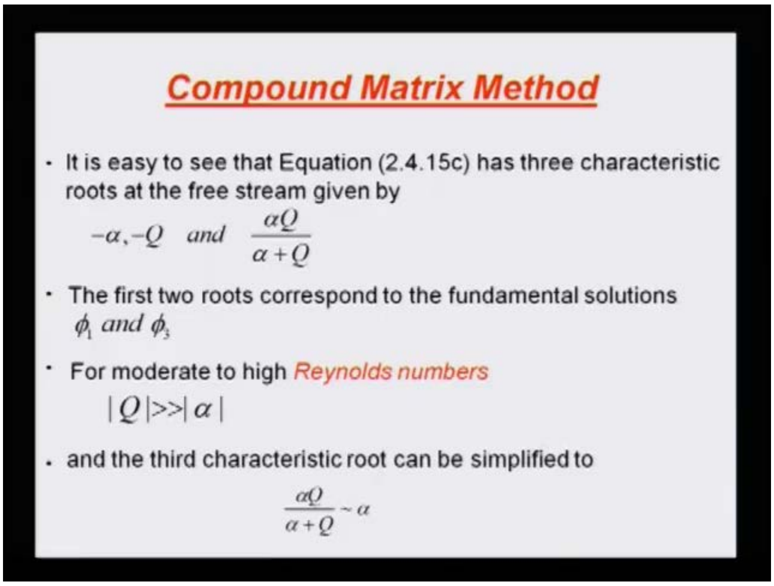

So, what happens is we can go there and work out all this 3 modes. Well, one thing is for sure you know, that you originally started with the exponent of minus alpha and minus Q. So, those two are there or it should be there. So, what you can do is, you can deflect this equation, remove those factors out. 5 plus alpha and 5 plus Q and then, you will see that the third root is like this. What is this third root? Looks like something, which we did not want to have and has come about. Is not it? Because, we want to have only those modes which decay with y and here is a mode which actually, cruse with y, if alpha real is positive. And well, it may look somewhat cumbersome, but you can reason it out that this mode Q is much greater than mode alpha. So, we can eliminate this, in comparison to Q and then, this Q, Q cancels and then, this factor actually goes like this.

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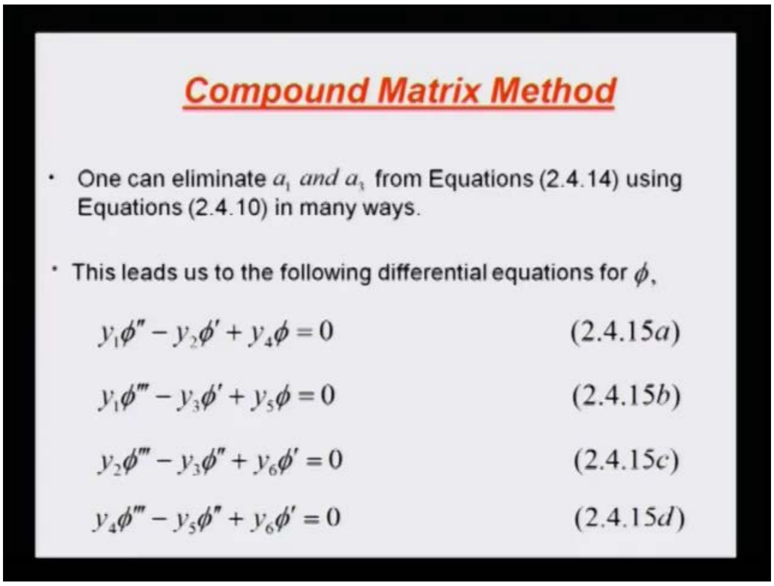

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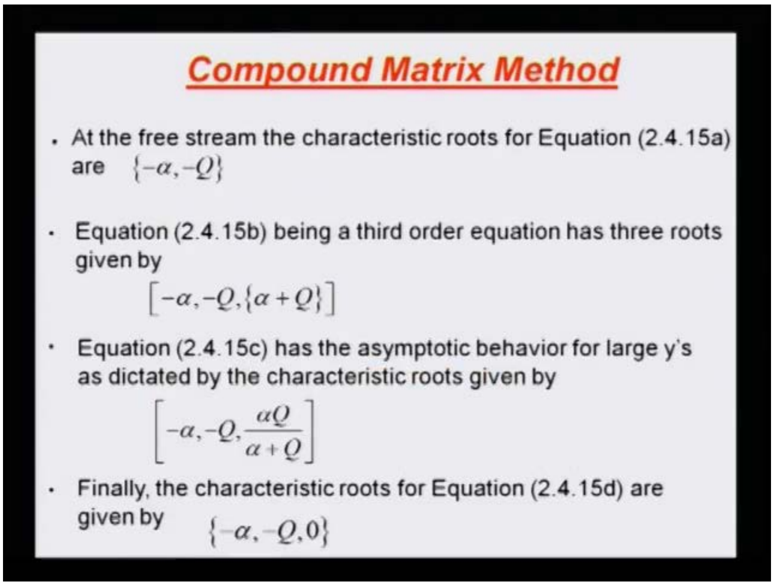

So, what happens is by being little careless, what we wanted to avoid, we have invited it back through the back door. So, we get the third mode which we did not want. It does not belong there. It grows with y. So, you see, this was such a simple thing. But, it took some time to realize that it should be so. So, what happens is, this third equation that we have identified here, that 15c is not something, I would like to double with, because it has a spurious mode. You can go through this exercise and do it for all the 4 equations. And what you would find, that corresponding to the first equation which is a second order ODE, you have 2 modes with the correct exponent of minus alpha and minus Q, that is what we want. So, this looks like a consistent equation. c was a not a consistent equation. You look at the second equation, which is also a third order equation. And it has 3 roots and this 3 roots, 2 of them are the ones which we are looking for, the third one is alpha plus Q. And this one we talked about, just now. c and that d is this, alpha minus Q and 0. That is a third order equation.

Good for ask, that one of the mode actually corresponds to exponent equal to 0. That means, that will not vary with y. That would be a sort of a some constant. So, out of this 4 combinations, you could choose. Which are the ones, that you could choose? The first one and the last one. The first and the last one are the ones that we can accept it, while the second and the third one, have spurious mode. This is really a violently growing mode. You should never touch b, because Q is large and you have a mode which is alpha plus Q. So, this will involve in the viscous unstable mode. And this one will be inviscid unstable mode, while this two are acceptable.

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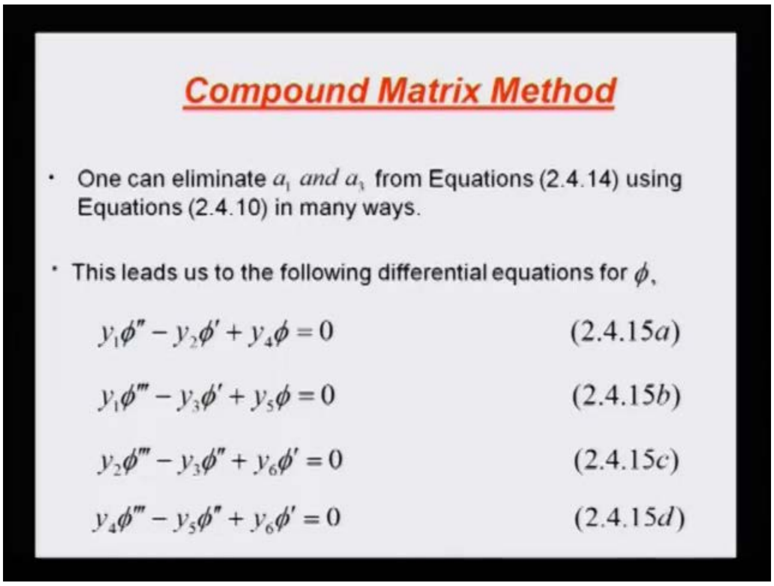

Now, this two are acceptable, but which one to choose? Let us look at those equations and decide for our self, which one to choose. You see what whatever you done so far, we have obtained this y1 to y6, by marching from free stream to the wall. Now, what we want to do is we want to the other way round. We will start because, we have the conditions given at the wall for phi. Is not it? What is the condition given at the wall? phi and phi prime is 0. Now, I have to choose between a and b. If I look at equation a, at the wall what it is? This will be 0, phi 0 phi prime is 0. What about phi double prime? It will be 0, how? We are talking about Eigen functions. So, if it is a Eigen function, what does it do? It satisfies the dispersion relation and what is the dispersion relation? y1 at the wall equal to 0. So, what we have here, phi double prime becomes of the form 0 y 0. Is not it? I can post these two term on this side and divided by y1. So, I will have a 0 by 0 form. So, it becomes kind of a indeterminate quantities.

So, you got to realize that. So, it is not very trivial to handle this. Now, whereas, if you look at the forth equation, you do not have such a problem, because this part will go to 0. And you will be solving this equation and it should be okay. So, in a sense, what I am suggesting to you, that if you are solving Eigenvalue problem, then it would be advisable to use the forth equation, rather than any other one. 2 are unstable, 1 is difficult to handle at the wall. Well, people have made all kinds of observations and saying that, if it is indeterminate at the wall, So what you do not do that?

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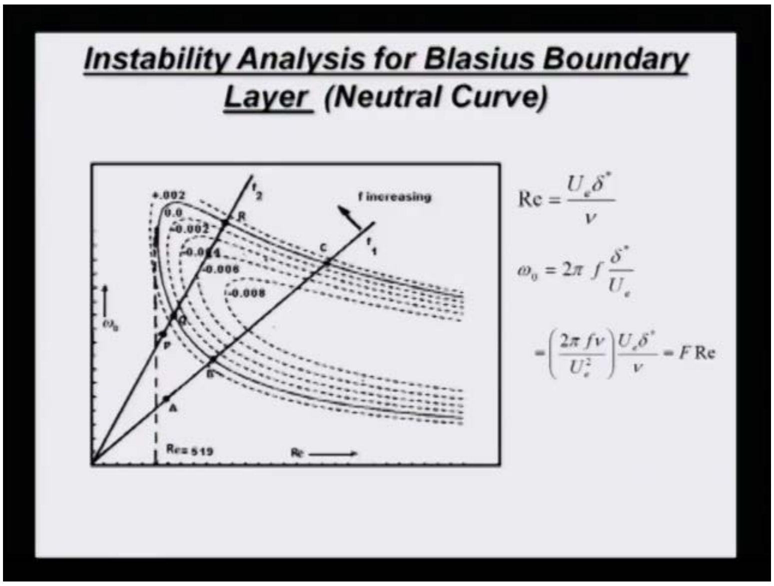

You would actually solve Orr-Sommerfeld equation itself, by 1 step or few steps. And then, you start off using the first equation. It is not elegant, because problem of stiffness, growth of parasitic error do remain. So, we have come to a stage, where we are at business. It took us about 60 70 years of hard work to come to a stage, where we could get some results. It is not that people at earlier times, did not have this results, they did. But accuracy of those results are in question, they have to do lots of extra work. You can do all kinds of procedure of Gram-Schmidt Orthogonalisation, etcetera. To do all that, that meant horrendous amount of work.

But, what you are looking here is very elegant, simple solution. And this simplicity has come about because you spend more time on the drawing board and less time in the computer. You have the other alternative. You do not want to work with compound matrix method, you will have to do lots of jugglery with computed solutions. So, what did we start looking at? We started looking at spatial instability problem. So, for a given Reynolds number, what we did? We fixed a frequency. Let us call that frequency in hertz as f. If I give the frequency, in hertz as F, what will be the dimensional circular frequency. In terms of radian per second, that will be 2 pi f, 2 pi f will be so many radians. So, f is what? Cycles per second, omega is radians per second. And if I want to non-dimensionalize that omega, what I should be doing? Radian has no dimension. Second is the only quantity. So, if I want to non-dimensionalize omega, I should multiplied by a time scale and what is the time scale? A time scale should be the length scale by the velocity scale.

So, omega naught is basically a non-dimensional circular frequency. This I could write it in this particular form. What I have done here? I have written there, 2 pi f nu by Ue square. So, that I have a factor here. Ue delta star y nu Ue delta star by nu is Reynolds number, a non-dimensional quantity. So, this quantity also, in parenthesis will be a non-dimensional frequency, which I call as capital F. So, what I have here, omega naught is equal to F into i. So, if I investigate the Re omega naught plane, a constant frequency lines would be, where F is equal to omega naught by Re. So, these lines originating from there, origin of this. R going to be a constant frequency. So, I think we will stop here. We have lots of things to talk about. This looks like a very simple diagram, but we can write a lot to explain the properties. All that, we had seen that frequency increasing, we are going to go this way. And what is important for one to understand that, there is a critical value below which, whatever you do any frequency you take, they are all going to be stable because the solid line indicates here, the neutral stability, inside you have a unstable region, outside you have stable region. I think, I will stop here. It needs a little more careful observation.