Instability and Transition of Fluid Flows
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Module No. # 01
Lecture No. # 01
AE-625
Transition and Turbulence

Good morning, welcome to this course on Transition and Turbulence AE 625, you can get my name, affiliation, contacts are given here. I would prefer for that, you can come any time that you wish to and just give me a phone call and just walk in.

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The title itself would suggest that we are looking at fluid flow transition - the instability is that lead the flow to turbulent straight, that is what is the main theme of this course. As we began today, let me try to tell you little bit about the scope of the course - the contents.

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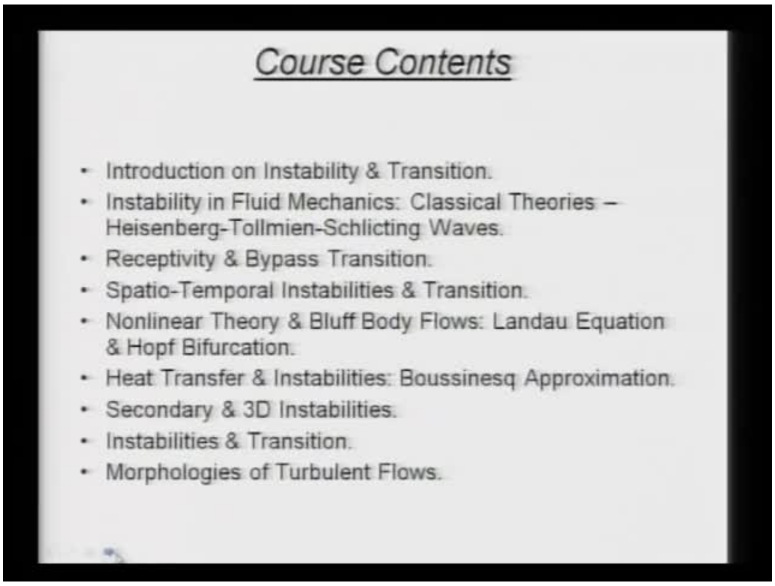

We will start off with the introduction on instability and transition that is what will begin today itself. We need to really look at how the way the subject has developed over the period, it was quite early on understood that, actual fluid flow behavior depends on flow instability. That means what? That, you have governing equations of motions, and those equations of motions are most of the time non-linear, partial differential equations are not solvable. In some specific cases, you could resolve them, and once you solve them, you would notice that you get some analytic solution and you go to the lab, perform the experiment, and you do not see them that is exactly what happened to strokes, sir GG strokes who is associated with the development of may be a Stroke's equation.

So, we solved flow past inside a pipe and try to compare this analytical solution with experimental data and it did not find anything. So, it does mean that, not all solutions are observable, we can see them. So, this is one thing that has a really triggered the tension of many people. So that is what we want to study in a instability of in fluid mechanics, because this is related to the instability of the solutions. What is the instability? That, if I have a solution or if I have a physical scenario, then if I… also have some background disturbances, which is not in my control, they do affect and instability implies the small imperceptible disturbances to create large effects, that is what we mean by instability, small cause leads to large disturbances.

There were classic theories developed, and you would be quite amazed if you have not heard of it before, that it was a one of the pioneer in physics who actually pick this problem up to solve, and he is no other than Heisenberg. As a student of Somerfield, he first started looking at flow instability, and subsequently the German school let by spangle and his students actually started looking at it, 2 notable students are Tollmien and Schlichting those of you have seen the book by Schlichtings. So, there is a same gentleman, they worked on it and what they found that the flow becomes unstable and then, you also see some waves.

So, this is what you mean by Heisenberg-Tollmien- Schlichting waves that we have written there, we will be spending quite a bit of time talking about that. What is important to realize that again those theoretical predictions of instability theories were not to be seen in experiment.

So, you can see that as junior by training or as a scientist by training, you cannot compartmentalize your activity, you cannot just simply say I am a theoretician I do not know, I do not care about experiments. The same way experimentalist cannot make that same claim that we do experiments if you do not see it then your theory is wrong, this happens all the time, it is a unfortunate business. And to tell you in this context also similar thing happen, when this three people one after the other started predicting these waves, it was nobody house other than professor JI teller of Cambridge he tried to experiment. And when he tried to perform these experiments, he did not see those waves, and so that led to a sort of a very big debate - international debate - a German school he is saying that there are waves and the english school led by JI teller there are no waves.

So, what happens? That is what is a big story, then came into the picture is this group from USA, Driedan and his colleagues, two of his colleagues are schuhbauer and skramstad, did perform some classic experiments at National Bureau Standard in Washington, they did those experiments and they were the first to observe those waves. And to perform those experiments that to work very hard, they realize that not all kinds of disturbances give raise to waves.

So, as a mathematician all us you would see like people talk about flow instability in terms of Eigen value problem, so Eigen value problem is what? We have a homogenous equation, homogenous boundary condition, and you try to get a solution out of it and those are your

Eigen solutions, so what does it mean actually? It means as if something is falling from the heaven, you are not putting any effort and you are seeing some results.

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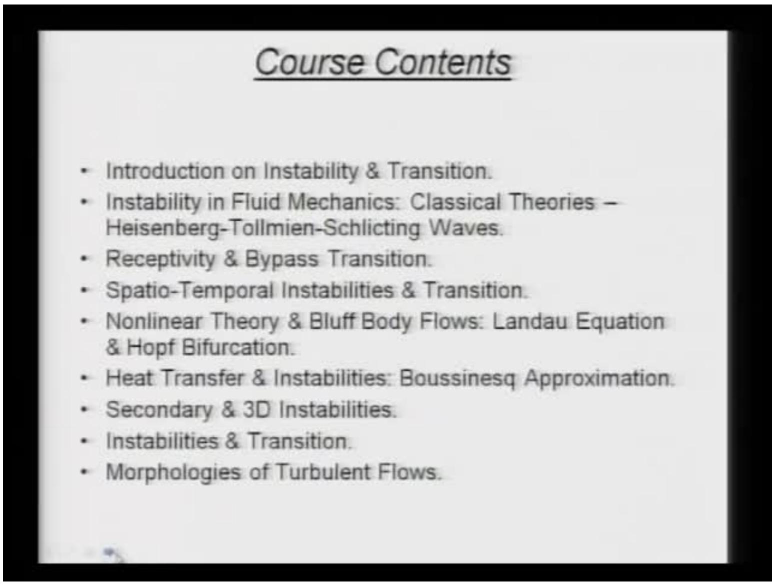

So, that is another drawback off Eigen value analysis. What one should instead look at, is connect the cause with the effect, so cause will be those background disturbances, they may not be all was measurable, but this still would have some quality, and that is what schuhbauer skramstad found out, Schuhbauer skramstad noted that not all disturbances produce waves. They could produce waves only when they vibrated a river inside the boundary layer. They found out that if you try to excite the flow with an acoustic noise, it does not create waves, and that is the subject that we talk about in this what we call as receptivity, that the flow is receptive to certain class of disturbances not all disturbances can give you instability waves.

So, if we are going to do that, we need to actually instead of studying stability, we should be studying receptivity, and that is a major thrust in this course, perhaps unique in all over the world that this subject is addressed in that framework, we do it a(( )). So, we will be talking about receptivity and what we would also notice that this experiment that was done by schuhbauer and skramstad required extreme care in setting this experiment up. We have to create a virtually noise free background reduce the disturbance as far as possible, so they actually designed a very very nice wind tunnel, that wind tunnel even 70 years afterwards continue to be used somewhere in some US universities.

So, we have to design an wind tunnel, you would find that there would be many experiment facilities they make, on substantiated claim that we have a very noise free tunnel, but they have no measurement of noise, though now well will not talk about that. What we are going to talk about there is a reality not the virtual one, where people make tall claims, you really have to design a tunnel where background disturbances have brought down to 0, and then, only we have to excite the flow deterministically like as I told you in schuhbauer skramstad experiment, that inside the boundary layer you started vibrating a river.

What happens if the amplitude of that vibration becomes very large? He do not see those waves, you do not see those waves, so what happens is, any transition, any instability and transition that takes the flow from a laminar state to terminal state without showing the waves have been historically called in the literature as bypass transition.

So, it bypasses the root of that Heisenberg – Tollmien - Schlichting waves this is what we would be studying. Then, people also have realized that there are a large number of cases, where your stability analysis shows the flow to be unstable, but in reality those flow also become turbulent. There are no flows which remain laminar forever, so please do understand that lamina flow is an exception; turbulent flow is a rule, unfortunately, bios in all the programs is to pretend as if laminar flow is everything and turbulent flow is a specialization, it is not true. People should know turbulent flow more than they should know about laminar flows, but even to know how the turbulent state appears, you need to know this process of transition from laminar to terminal that is precisely what we are doing in this code. What happens is as I told you that there are many flows situations where you would not see these waves, it is bypassed, etcetera.

There are also cases where the instability theory even says that it would not become unstable, and nothing more to exemplify this it is a case of that is a pi flow, theoretical it is shown to be stable, but we are low that a reynolds number above 2000 based on diameter of the pipe flow becomes terminal. Then, there is a quiet flow shared river, if I have flow between 2 parallel plates on the top plate is moving that is what we call as a quiet flow, quiet flow is also theoretically found to be step right. However, those flows are not really stable, so people have try to study one of the mechanism, people talk about some kind of spatio-temporal instabilities that may be affecting some of these flows, and this is what we are trying to look at ok.

This is about wall bounded flows, external flows that we have been talking about, because most of us I have a background of aerospace engineering, so we are more interested in external flows. But then, we also at times have to worry about not stream line shape, we have to worry about bluff shape. Think of an aircraft when the landing gear comes out, it is a flow past a cylinder, well, there are many such occasions you would see there are cavities etcetera.

So, there what happens is, we do not have streamline body flows, instead we have bluff body flows, and they also suffer some kind of an instability ok. Now instability parse instability per say can also occur in two different ways or a combination of these two ways, what are these two ways? The disturbances can grow in space, disturbances can grow in time, and disturbances can also grow in space and time that is what we talked about spatio-temporal instability, disturbances which simultaneously grow in space and time.

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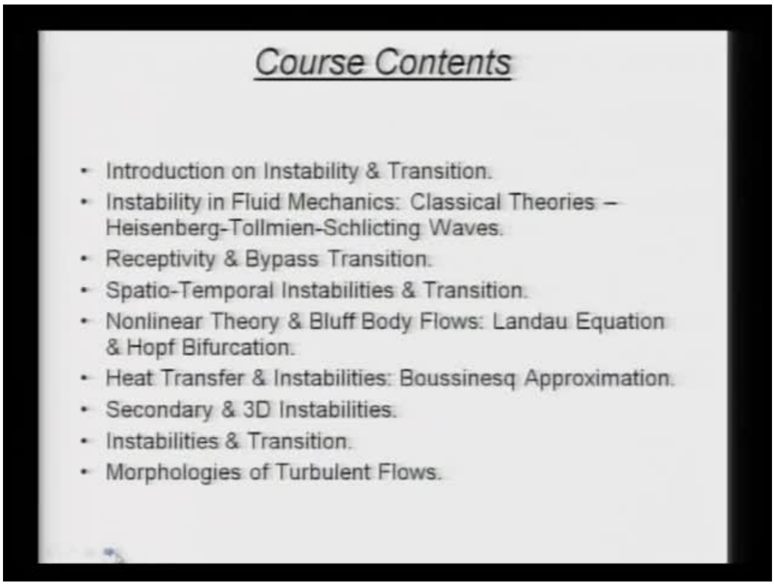

Now, this earlier part, external flows that we talked about pass streamline bodies, one of the characteristic feature people have noticed over the years, is at there the disturbances actually convict, as they grow that means, it is a spatial growth - growth in space ok. In contrast to that, let say flow past a cylinder is looked at, a bluff body flow; there you notice that initial is disturbances grow in time, if your position in the way you will look at it, you will see it grows in time.

Now, it is a interesting thing that is basically a sort of a evidence of instability, the disturbances are growing, but nonlinearity plays a very different role for this external flows or passed a streamline body and a bluff body. For a streamline body what happens? The nonlinearity actually accentuates – increases - the instability that is where you go. Please do not understand or make this misconception that flow becomes unstable or boom it becomes stable, it does not happen that way.

The instabilities grow, then that disturb flow further can become unstable, so the first instability will call it as a primary instability, the subsequent ones will call it as secondary instability, tertiary instability, and so on so forth. So, for a stream line body flows, the primary instabilities are predicted very nicely by those classical theories, while the secondary and the tertiary instabilities are due to some kind of non-linear effects and they actually accentuate the instability. While for a flow positive bluff body will show that a primary instability is a temporal instability, and nonlinearity here plays a very interested role, nonlinearity here actually moderates the primary instability.

So, the waves keeps growing, but then it saturates, we will see this, we will spend lot of time doing this, so these are some of the interesting things that we would be talking about, and you have heard of Landau; Landau came out with a equation which is called Stuart Landau equation. That tells you how this primary instability saturates into a non-linears action into another almost neutral amplified wave.

So, you start off from one equilibrium state that was your laminar flow, it became unstable, because of temporal instability, the nonlinearity saturates it, so you actually get a time periodic flow, this is what you see as the vortex shading behind a cylinder, that is a classic example, that you have a vortex shading - carbon bener vortex shading - behind a cylinder you say that they are very periodic, it will not just simply exploit that happens due to this non-linear action and landau actually worked out the equation for it, and we will also talk about bifurcation, what is bifurcation?

Now, we are talking about instability of flows, as I told you flow inside a pipe, the classical linear theory says, it is stable, but if I perform experiment I find that flow cannot be kept easily laminar if the Reynolds number is above 2000, if it is above 2000, then we will have to make some effort additional effort to keep the flow laminar, but if your Reynolds numbers are less than 2000, then even if you create lot of disturbance in that flow, it is still remains laminar.

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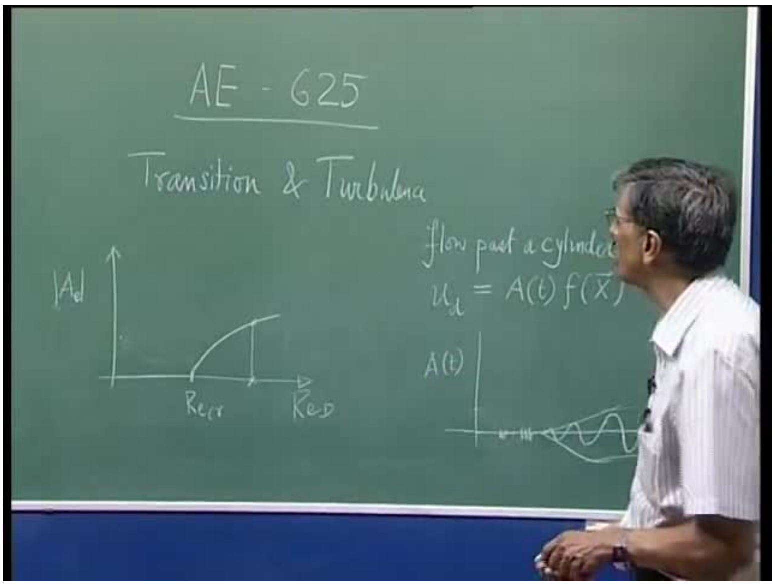

So, it seems the Reynolds number works like a kind of a parameter for the problem, and you have a critical value below which it remains stable, above which it is unstable. So, this kind of scenario where actually, we may look at, let say, flow past cylinder if I have were to be talking about on this axis I will be plot and say Reynolds number based on the diameter, and on this side, let me just simply plot the amplitude.

So, basically what we are talking about that flow past a cylinder we are going to get some disturbances, so this let me write it as Ud subscript d implying disturbance field, on that I will write it as some A of t, I told you it suffers from a temporal instability, so let us called that if we have t and that would be multiplied by some function of space f of X right. Now, this is that, a that we have plotting, the time dependent what do you find that up to some reynolds number that up to that, the flow remains laminar, that means what? This amplitude does not die, so even if you create some disturbance that disturbance will eventually decay.

So, this is something like your equilibrium flow, what do you mean by this, so what we can do is, if I plot A of t versus t I may get something like this, that initially let say I create some kind of a disturbance, and if I am below this Re critical value, then what we will happen? It may just simply go on, go on, decay that is subcritical flow.

So, this is the subcritical part, and this part I will call it as supercritical. So, this is your subcritical solution, where you may have initially created some disturbances at t equal to 0, but eventually decays. Whereas in case of a supercritical case, what happens is, something different happen, there what we will find that, suppose I start off with some virtually no disturbance at all, but there are background disturbances in the experimental facility, then what will happen for a supercritical scenario? I would have something like this, I will show you detailed results of this theoretical computational as well as experimental, people have done it, and they really find that something very interesting thing happen, it remains virtually like this then you actually see some kind of a very high frequency oscillations; once in a while and then it slowly takes up.

So, this is what we meant by bluff body flow instability, and then what happens is, it just saturates in an envelope and this growth of the amplitude curve is what is of interest. So, in this supercritical case what happens is, you are going to see that starting from Re critical, this is your equilibrium state, so this is my A e that we are talking about, this is that A e, the amplitude, it is 2 times it is a periodic oscillation, so it is 2 times A e, so I can plot that, and what you would find that the scenario is like this, up to R e critical that equilibrium amplitude remains 0, and then it actually goes like this.

So, this behavior is typical of bluff body flows, and this transition from a subcritical to supercritical state is what is often called as bifurcation. Now, why do call the solution bifurcation? It means that in the supercritical stage I can actually get a solution which could be here, or if I am carefully doing the experiment I can also assume it (( )) case.

For example, for the pipe flow experiments we will talk about today itself time permitting, Osborne Reynolds did those experiments, we talked about just now a Reynolds number of 2000 being the critical Reynolds number. Osborne Reynolds number carefully it is because experiments and he could keep the flow laminar all though we have to 12830 ok - nothing to be surprised about because, later on people even did experiments and created pipe flow which was stable for Reynolds number hundred thousand. So, what it means, that your solution bifurcates from this point onwards, here you will only have one solution, but here you will have multiplicity of solution, wow, this was a typical attribute of systems which suffer temporal instabilities specially flow past bluff bodies, you can think about.

So, this kind of bifurcation is what is called as Hopf bifurcation, well there are many types of bifurcation; Hopf bifurcation is one of it, so we will use that. And we will also talk about the other interesting things like, effect of heat transfer around flow instabilities this is a very important issue, because if we are talking about, let say, flow past flow in our atmosphere, the weather system, here what you have, you cannot just simply talk about the instability of the atmosphere only in the absence of heat transform.

The heat transfer as it occurs, sun is our main source of energy, but these days we are also creating a lot of heat or Sulfur anthropogenic heat transfer, manmade heat how it affects the system dynamics. You got to understand that in this course we are going to take very deterministic approach of systems in studying there instability.

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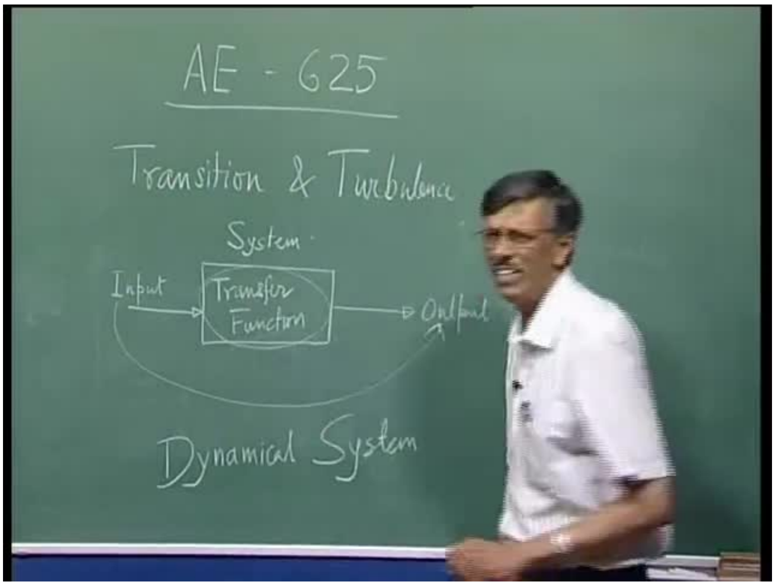

So, what we are going to talk about is, basically a system which I would represent by let say a black box, so this is your system, now what happens to the system, the system is bombarded by input, we like it or not there is there, that is what we have been studying, and that we get the output. And we have already seen the talking about receptivity is basically trying to connect input with the output through the system dynamics, what is the system dynamics, for the time being I will call that as the transfer function, so what does this system do? It takes the input multiplies it by the transfer function to give you an output, transfer function is done, the property of the system.

So, I can have flow in the weather system without though heat transfer, I am talking about one kind of transfer function, the moment I add the heat transfer a transfer function has change, this is what I was telling you also about Reynolds experiment, it something to change the transfer function or he did something to reduce the input.

So, they are lots of very, very, interesting thing in studying any dynamical system, this basically is one of the goal of this course that we talk about systems in nature has a dynamical system, talk about the economics of a country, it is a dynamical system. We do not know how to model it, cannot get it is transfer function currently that is a different issue, but hopefully in future, we should be able to do that, look at all those Smart Alex in the finance field, they actually play around find out how this micro fluctuations in the input and they converted into dollars in the pocket, that is also they have study, they used kiyos dynamics.

So, we talk about this, any practical system tells you that this transfer function did not always been very deterministic, a very good example is, as I always fond of quoting, it is tossing a coin we cannot even predict it is transfer function, why? have in that is a part follows of a part physical, but the fact is we do not know what are the players that determine the outcome or the same way economics as a subject we do not know there are too many contributing factors that makes the study of the dynamical system very, very, difficult, you cannot get a deterministic portraitor, we have to talk about it has a stochastic system and that happens is, stochastic system means what? It is probabilistic, but also time dependent that is stochastic right.

Tossing a coin could be a probabilistic event, you do not know whether it is stochastic or not, because if I am doing the experiment in this room tossing a coin, depending on how the temperature inside the room is changing with time, etcetera, or if I keep a windows open or something and the earth drift saying that can all effect the outcome of the experiment.

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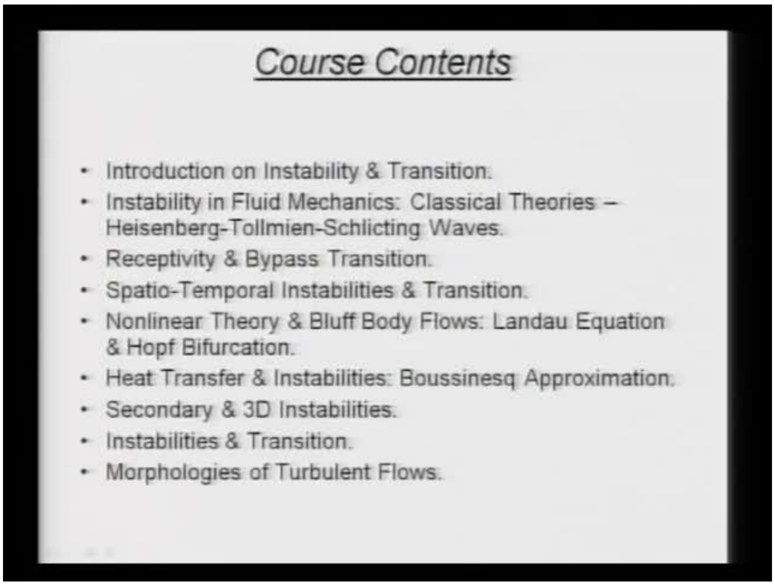

So, whether it is simple probabilistic time independent or it is time dependent that is stochastic, we do not know. So, basically in terms of instabilities as effected by heat transfer is a about the subject that gives us a some glimpse of what may happen to a complex system we will talk about that, and then I will talk about secondary and three dimensional instabilities back to our aerospace applications. Where you will see how this secondary and the 3 dimensional instabilities come into play, and I told you very clearly about this aspect instabilities and transition, instability does not mean that you would get a transition immediately.

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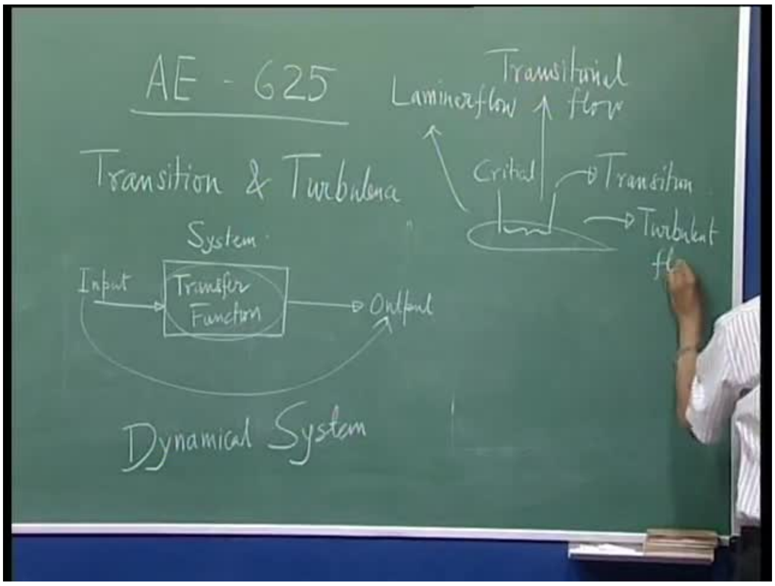

So, basically instability and transition are not synonymous, you have a finite region over which the flow becomes unstable. So, if I am studying a flow past an aero foil, so the flow can become critical at this stage, but if I am looking at, say, fully turbulent state it may have happened here. So, this is what I may call as a critical point and this is well, let say finally, the transition takes place well there are very as definitions of transition forty five able.

So, let say we adapt to all of those and that says it is there, so this is a very nontrivial space, so this is what we will be calling it as the transitional flow. So, on this side we have laminar flow and on this side we have turbulent flow and these two are bridged by transitional flow. So, it is so happens that this region over which transition occurs is not trivial, a very good example would be flows in a turbine, in a gas turbine, in a turbine flow accelerate. Accelerate means it is under the influence of favorable pressure gradient, while that turbine also sits downs stream of the combustion chamber and combustion through all this chemical reaction makes the flow very, very dirty and very, very, noisy.

So, you have a competing dialog going on, you have a dirty flow coming in bombarding and you are imposing an acceleration which is trying to moderate, that makes this region very, very, significantly large. So, to understand transitional flow is a very important issue and that is what we need to really keep aware of, and once you come to the turbulent state, you need to know what is really turbulent flow and I would – if time permitting - I will talk about this morphology, what constitutes? Then, this course we are not doing turbulence modeling, on we would keep that aside, we will focus in the scientific aspect of it. Well, we talk about it, see basically, all we want to do eventually try to understand what is turbulent flow.

So, we have taken a different route now to understanding this, we are coming from the laminar flow side and see where we are. Our point of view is that the turbulent state where we have to arrive would be determined by the process of transition, it is not unique. Now, we have seen what our course of content is going to be like, this is something you must, we are also, curious to know what the references that we are going to use. Well, we are going to use this book, it is a book that myself and Dr. poinsol have written, will not do the chemical reaction path, we will just do this part in stability of flows with and the outer transfer.

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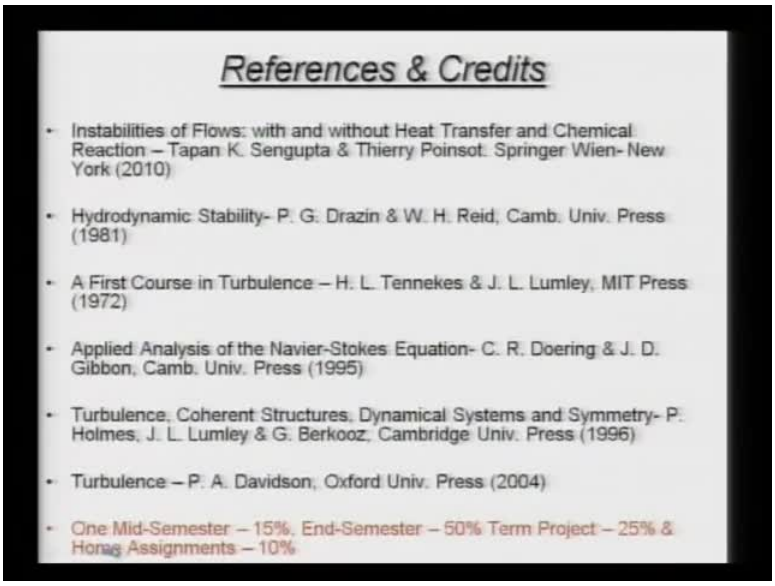

So, this book is available, apart from that book, I would recommend that one look at this book by Drazin and Reid, it is now quite a classic book titled hydrodynamic stability, and this two will be more than adequate, there are many other books, you can take a look at them, but it is not necessary, so will stick to these two books only and that should be adequate. And in the turbulent flow path, we have a very nice book here, first course in turbulent by Tennekes and Lumley that is one book that we would be using in bits and pieces. But for understanding a buffology of turbulent flow, you would also be looking at this applied analysis of the Navier-Stokes equation by Doering and gibbon; this is a very short model graph, but very nice book. And as I seen and discussed that we like to study the flow as a dynamical system and we try to find out what it is transfer function etcetera is.

So that is covered very nicely in this book called turbulence coherent structured dynamical systems and symmetry by Holmes and Lumley and Berkooz they have written this exceedingly nice book. Our interest is to basically characterize turbulent flow by some diagnostic tools of dynamical system theory. One of the dynamical system theory tools that we are going to use time and again is proper orthogonal decomposition. Let me tell you the good news that that tool was developed by professor D D Kosambi of India, despite all the things that you would read in literature, it was by Kosambi would read that published in a paper Indian general in 1943 karron and (( )) and all other people came later.

So, there is something we are going to use the POD as a tool proper orthogonal decomposition and trying to understand, you see, turbulent flow may look kyotic, but still within that kyos also it still you would see some pattern, those patterns are what are called as coherent structures, so this was attempted. So, POD as a tool allows you to project that stochastic system into a deterministic basis and see whether you can pick up those coherent structure, that is what we will go into spend quite a bit of time, in fact, many of the students work with us, with me, they do use POD as a tool and we have done some very, very, interesting thing in recent years using POD.

So, we will do that and there is also this book by professor Davidson, title turbulence, it is a fairly and recent book and you would find it interesting, there is a something that would be of interest to you that we will only have one (( )) we will have a comprehensive endson, which will cover the whole course and I will ask you to do instead of a in term project etcetera, that would be 25 percent and this more home assignments your regularity etcetera, we will take care of this less 10 percent, so will be your following this and this is the way that we would doing.

Now, this we already have started we have told you but still from the less, we see how the subject of transition has developed overly years. Now, I told you about Reynolds experiment, but it was not Reynolds who started this investigation, rather I mentioned to you about those pipe flow experiments means calculations done by stokes. And his inability to see those experiments, experimental values obtains by his theoretical analysis. What was the anomaly simply stokes was looking at his laminar flow calculations, while the experimental results for the pipe flows they are for turbulent flows, people were thinking what is happening and what is this turbulent state.

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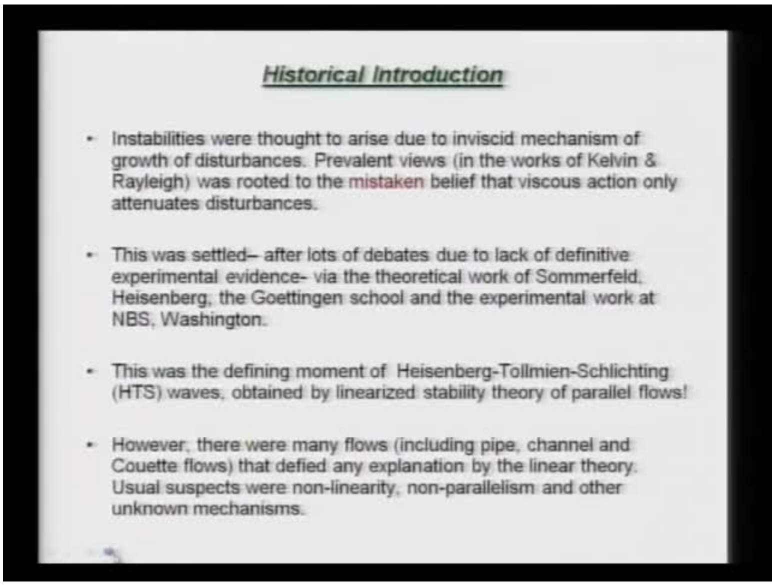

So, that really set into motion lot of work, that is embedded in work of Kelvin, Rayleigh etcetera, they understood that instabilities are due to growth of disturbances. That is very fine and that is what we are all talking about, but they make a cardinal mistake, they said it is an inviscid mechanism. Rayleigh even found out an equation to show how an inviscid mechanisms comes about, he gave some theorems and criterions to find out where flow becomes unstable.

Now, they also made this observation that why one should look at the inviscid mechanisms? There point of view was very simply this, that if discuss action is there it is dissipative, so if there is some kind of a dissipative mechanism that will actually damp out those disturbances, so it is perfectly alright for us to study the inviscid mechanism, because if there are any viscous this actions, they will only attenuate disturbances.

This looks very convincing compiling to follow, but then there were other people mathematician and (( )) who were not thoroughly convinced, two of them this mathematician by name or and (( )), then Somerfeld they actually wrote down the disturbance equations including the viscous action and that equation is called the or Somerfeld equation. This or Somerfeld equation is a central piece today, but in those days nobody thought that was necessary, because I told you that the viscous action only attenuate a disturbance.

So, there are absolutely no need for adding in complication through the viscous action, despite that has Heisenberg under the guidance of somerfeld (( )) disicipation and it did try to say what could happen? That thesis was very unique, the examination commentary look at the thesis they could not find any mistake, but they did not also believe it, and that is the story of how quantum mechanics came into way. Heisenberg after his thesis stopped working on fluid flow and he went on to establish quantum mechanics. So, you understand that, this subject has a well checked history, while somerfeld was a brilliant researcher, four of his student received a noble prize he never got it, so that is another story.

Subsequently, also content, well, along the same time ludwick, prandal and his students also where interested looking at this instability problem, and I told you about those experiments, sorry, those calculations done by Tollmien- Schlichting, they did come out with some results, but those were again negated by G I Taylor's experiment in Cambridge. He never could found, well because Taylor did not read those results, analytical results very carefully, they predicted way for certain frequencies and in the Taylor's experiment he used a sort of a bump - oscillating bump - on flat plate, but the bump was oscillating at a wrong frequency, low frequency, and believe me in almost in 95 and 96 we explain really what happen in this Taylors experiment, so it took another 60 years to come out with, it was done here by one of the couple of our students.

So, if you try to excite a flow at a lower frequency and you do not see it, do not kill the messenger, unfortunately everybody did, but then around the same time Dryden and his group at a National Bureau of Standard did those experiments that we already discussed, that schuhbauer skramstad experiments, they triggered out that to investigate and obtain those waves, we will have to remove the background disturbances and then, give some kind of a deterministic disturbances of finite amplitude, then you dynamical system picture is quite nicely constructed. We have a very definitive input and the laminar flow that is your transfer function, and you try to find out what is there output going to be, those disturbance growing and that actually help the subject tremendously, those classic experiments done by schuhbauer skramstad.

So, this was really the defining movement and despite what is written in any book and many books I would always refer to it as Heisenberg-Tollmien- Schlichting waves. So, I will call it as HTS waves, but you will find in most of the literature they talk about T S wave, we should give Heisenberg is correct in fact, he was one of the pioneer in this field.

They obtained all those waves using a linearized stability theory, they made some assumption of parallel flows, despite that they did predict those waves and schuhbauer skramstad found those out in experiments, so this was a glorious of period. And I mentioned to you also but they were many flows; pipe flows, channel flows, quiet flows, that did not explain the flow instability by the same linearized stability theory develop there.

So, we are happy to give excuses right, so people gave excuses that may be these are the suspects, nonlinearity because it was a linear theory or may be some non-linear mechanism is taking place, than the flow was considered parallel in this theory - parallel means what is streamlines a parallel – that you know, a boundary layer grow so the they do not remain parallel. So, that is that, growth part is important non-parallelism and maybe there are other unknown mechanisms.

So, even today, a large number of us try to spend time finding out, so new unknown mechanisms. So, there are lots of such activities that go on, but it was realized, so I think I will stop here and we will start from here in the next class.